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## **THE HEBREW UNIVERSITY OF JERUSALEM**

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### **A NOTE ON THE IDENTIFICATION OF PRESENT BIAS AND FORGETTING FROM TASK-COMPLETION DATA**

By

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# A Note on the Identification of Present Bias and Forgetting from Task-Completion Data\*

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## Abstract

Recent work highlights that identification of present bias using task-completion data is problematic. In this note, we add to the literature in two ways. First, whereas prior work considers single-deadline tasks, we consider tasks for which there is a series of deadlines, with incremental penalties for missing each deadline. Second, we also consider identification of forgetting from the same type of data. Using numerical examples, we demonstrate that identification of present bias is still problematic even when there are multiple deadlines. Identification of forgetting perhaps holds more promise theoretically, although in practice we suspect it too is problematic.

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# 1 Introduction

Recent work by Heidhues and Strack (2021) and Martinez et al. (2017) addresses whether one can identify present bias from data on task completion in a fixed environment. Specifically, both papers consider a fixed environment with a single deadline: a task must be completed by some deadline, with a penalty (possibly infinite) if it is not. They further assume that there is day-to-day variation in the (immediate) cost of carrying out the task, and thus there is reason to delay so as to complete the task on a low-effort-cost day. The identification question is whether one can separately identify present bias and the effort-cost distribution.

In a theoretical analysis, Heidhues and Strack (2021) prove that, even if one can observe the exact stopping probability in each period, one cannot separately identify present bias and the effort-cost distribution unless one imposes some structure on that effort-cost distribution. In an empirical analysis, Martinez et al. (2017) attempt to estimate present bias using income-tax-completion data, where they assume different functional forms for the distribution of effort costs and assess how sensitive the estimates are to those assumptions.

In this note, we add to the literature in two ways. First, we consider a multiple-deadline task: a task must be completed, but instead of there being a single deadline, there is a series of deadlines, with incremental penalties for missing each deadline. Such situations are common—for instance, in the domain of New York City parking tickets studied in Heffetz et al. (2022). Moreover, the constructive proof used by Heidhues and Strack (2021) does not directly extend to this domain. Hence, we study to what extent the existence of multiple deadlines can aid in identification of present bias.

Second, we consider the possibility of forgetting, which has also been studied as a reason for delay in task completion (Holman and Zaidi (2010), Taubinsky (2014), Ericson (2017), and Altmann et al. (2022)). We study to what extent one can use task-completion data to separately identify forgetting and the effort-cost distribution, again within a multiple-deadline environment.

In this note, we do not provide any formal proofs. Rather, we shed some initial insight on these questions using numerical examples. These numerical examples demonstrate that identification of present bias is still problematic even when there are multiple deadlines.

Identification of forgetting perhaps holds more promise, in part because forgetting can generate patterns that cannot emerge from a model without forgetting. Nonetheless, we argue that, given the type of variation likely to be available in actual datasets, in practice it too is problematic.

## 2 Model

We consider a variant of the McCall (1970) job-search model. It is similar in structure to that used in Heidhues and Strack (2021), except that we permit multiple deadlines and forgetting.

### 2.1 Environment: A Multiple-Deadline Task

We consider a fixed environment with multiple deadlines. On each day  $d \in \{1, 2, \dots\}$ , a person decides whether to complete a task. There is a series of  $N$  deadlines,  $D_1, \dots, D_N$ , where missing deadline  $D_n$  leads to a monetary penalty  $a_n$ . Hence, if for instance there are three deadlines ( $N = 3$ ), the net monetary penalty that must be paid as a function of the day  $d$  on which the person completes the task is

$$A_d \equiv \begin{cases} 0 & \text{if } d \in \{1, \dots, D_1\} \\ a_1 & \text{if } d \in \{D_1 + 1, \dots, D_2\} \\ a_1 + a_2 & \text{if } d \in \{D_2 + 1, \dots, D_3\} \\ a_1 + a_2 + a_3 & \text{if } d \in \{D_3 + 1, \dots\}. \end{cases}$$

The person seeks a convenient (low-effort-cost) day to complete the task. Let  $c_d$  denote the effort cost on day  $d$ , drawn i.i.d. from distribution  $F$ . The person knows  $F$  in advance, and learns the realization  $c_d$  on day  $d$  before deciding whether to complete the task that day. Hence, the total cost of completing the task on day  $d$  includes both the monetary cost  $A_d$  and the effort cost  $c_d$ . We assume that the effort cost  $c_d$  is experienced on day  $d$  (i.e., it is effort exerted now), while the monetary cost  $A_d$  is experienced on day  $d + 1$  (i.e., it requires forgone future consumption). For the examples below, where we make the natural

assumption (given daily decisions) of  $\delta = 1$ , it is irrelevant when exactly in the future  $A_d$  is experienced.

The person seeks to minimize her expected discounted total cost, where the person has  $\beta, \delta$  discounting as in Laibson (1997) and O'Donoghue and Rabin (1999). Specifically, if  $\gamma_{d'}$  is the (monetary or effort) cost incurred on day  $d'$ , the expected discounted total cost from the perspective of day  $d$  is  $\Gamma^d \equiv E [\gamma_d + \beta \sum_{d'=d+1}^{\infty} \delta^{d'-d} \gamma_{d'}]$ . For instance, if on day  $d$  the person completes the task at an effort cost  $c_d$ , then  $\Gamma^d = c_d + \beta \delta A_d$ . Alternatively, if on day  $d$  the person commits to complete the task on day  $d' > d$ , then  $\Gamma^d = \beta \delta^{d'-d} E(c_{d'}) + \beta \delta^{d'+1-d} A_{d'}$ .

$\beta, \delta$  discounting permits both standard exponential discounting (captured by  $\delta \leq 1$ ) and time-inconsistent present bias (captured by  $\beta \leq 1$ ). If  $\beta < 1$ , it matters what the person believes about her own future present bias. Following O'Donoghue and Rabin (2001), we let  $\hat{\beta} \in [\beta, 1]$  denote the person's belief about her future present bias. With this formulation,  $\hat{\beta} = \beta$  implies the person is fully *sophisticated* and has correct beliefs,  $\hat{\beta} = 1$  implies the person is fully *naive* and believes she'll have no future present bias, and  $\hat{\beta} \in (\beta, 1)$  implies the person is *partially naive*.

The person might also forget about the need to complete the task. On day  $d$ , the person can be in one of two states,  $s_d = Y$  or  $s_d = N$ . The state  $s_d = Y$  represents that the task is on the person's mind, in which case the person actively decides whether to complete it. The state  $s_d = N$  represents that the task is not on the person's mind—i.e., she has forgotten about it—in which case the person necessarily does not complete the task.

The day- $d$  state  $s_d$  depends on the day- $(d-1)$  state  $s_{d-1}$  according to  $\Pr(s_d = Y | s_{d-1} = Y) = \lambda^Y$  and  $\Pr(s_d = Y | s_{d-1} = N) = \lambda^N$ . This structure nests several special cases. First,  $\lambda^Y = \lambda^N = 1$  is the case of no forgetting. Second,  $0 < \lambda^Y = \lambda^N < 1$  is the simple case where there is an i.i.d. probability of remembering on each day. Third,  $0 \leq \lambda^N < \lambda^Y \leq 1$  is perhaps the main case of interest where the likelihood of thinking about the task today is larger if the person also thought about the task yesterday. We assume an exogenous probability  $\Lambda_1^Y$  that the task is on the mind on day 1.

This model of forgetting is similar in structure to that used in Holman and Zaidi (2010), Taubinsky (2014), Ericson (2017), and Altmann et al. (2022). An important issue highlighted in that literature is whether the person is aware versus unaware of her future propensity

to forget. We let  $\hat{\lambda}^Y$  and  $\hat{\lambda}^N$  denote the person's beliefs about her future  $\lambda^Y$  and  $\lambda^N$ . Hence,  $\hat{\lambda}^Y = \lambda^Y$  and  $\hat{\lambda}^N = \lambda^N$  implies full awareness and understanding of future forgetting,  $\hat{\lambda}^Y = \hat{\lambda}^N = 1$  implies full unawareness, and  $\hat{\lambda}^Y \in (\lambda^Y, 1)$  and  $\hat{\lambda}^N \in (\lambda^N, 1)$  implies partial awareness.

Finally, we close the model by assuming that if a person delays beyond (the last) deadline  $D_N$ , there is an exogenous continuation cost  $Z \equiv a_1 + \dots + a_N + z$ . In words, this continuation cost includes the monetary penalties  $(a_1 + \dots + a_N)$  as well as all expected effort and other costs that might occur in the further future ( $z$ ).

## 2.2 Solving for Behavior

Consider first the case of full awareness about both present bias ( $\hat{\beta} = \beta$ ) and forgetting ( $\hat{\lambda}^Y = \lambda^Y$  and  $\hat{\lambda}^N = \lambda^N$ ). On each day  $d$ , there will be a cutoff cost  $\bar{c}_d$  such that a person with  $s_d = Y$  completes the task for any  $c_d \leq \bar{c}_d$ , and thus the probability that a person with  $s_d = Y$  will complete the task on day  $d$  is  $F(\bar{c}_d)$ .<sup>1</sup>

One can solve for the  $\bar{c}_d$ 's by working backward. Specifically, the  $\bar{c}_d$ 's for all  $d \in \{1, \dots, D_N\}$  can be derived recursively using the following equations:

$$\bar{c}_d = \beta\delta [W_{d+1} - A_d] \quad \text{for all } d \in \{1, \dots, D_N\} \quad (1)$$

$$W_{d+1} = \lambda^Y W_{d+1}^Y + (1 - \lambda^Y) W_{d+1}^N \quad \text{for all } d \in \{1, \dots, D_N\} \quad (2)$$

$$W_d^Y = F(\bar{c}_d) [E(c|c \leq \bar{c}_d) + \delta A_d] + (1 - F(\bar{c}_d))\delta W_{d+1} \quad \text{for all } d \in \{1, \dots, D_N\} \quad (3)$$

$$W_d^N = \delta [\lambda^N W_{d+1}^Y + (1 - \lambda^N) W_{d+1}^N] \quad \text{for all } d \in \{1, \dots, D_N\} \quad (4)$$

$$W_{D_N+1}^Y = W_{D_N+1}^N = Z \quad (5)$$

In these equations, the  $W$ 's denote various "long-run" (without  $\beta$ ) expected continuation costs. First,  $W_{d+1}$  represents expected continuation costs starting on day  $d + 1$  for a person on day  $d$  with  $s_d = Y$ . Hence, on day  $d$ , a person with  $s_d = Y$  will complete the task

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<sup>1</sup>While the equations assume that the person completes the task when indifferent, in our numerical examples there is zero probability of being indifferent.

when  $c_d + \beta\delta A_d \leq \beta\delta W_{d+1}$ , from which equation (1) follows. Next,  $W_d^Y$  and  $W_d^N$  represent expected continuation costs starting on day  $d$  conditional on  $s_d = Y$  and  $s_d = N$ , respectively. Equations (3) and (4) derive these from the  $\bar{c}_d$ 's, and then equation (2) uses them to derive  $W_{d+1}$ . Finally, equation (5) reflects our assumption that if the person delays beyond the last deadline  $D_N$  then the cost will be  $Z$ .<sup>2</sup>

Finally, whereas  $F(\bar{c}_d)$  is the probability that a person with  $s_d = Y$  will complete the task on day  $d$ , data would contain only the unconditional (without knowing the person's state  $s_d$ ) probability. Let  $\Lambda_d^Y$  be the likelihood that a person who has not completed the task before day  $d$  has  $s_d = Y$ , where  $\Lambda_1^Y$  is exogenous. Then for all  $d > 1$

$$\Lambda_d^Y = \frac{\Lambda_{d-1}^Y(1 - F(\bar{c}_{d-1}))\lambda^Y + (1 - \Lambda_{d-1}^Y)\lambda^N}{\Lambda_{d-1}^Y(1 - F(\bar{c}_{d-1})) + (1 - \Lambda_{d-1}^Y)}. \quad (6)$$

The unconditional probability of completing the task on day  $d$  is  $h_d \equiv \Lambda_d^Y F(\bar{c}_d)$ . Equation (6) is relevant only when there is forgetting; if there is no forgetting ( $\lambda^Y = \lambda^N = \Lambda_1^1 = 1$ ), then  $\Lambda_d^Y = 1$  for all  $d$ , and thus  $h_d \equiv F(\bar{c}_d)$ .

Again, everything above assumes full awareness about both present bias ( $\hat{\beta} = \beta$ ) and forgetting ( $\hat{\lambda}^Y = \lambda^Y$  and  $\hat{\lambda}^N = \lambda^N$ ). If there is unawareness on either dimension, then we must distinguish perceived continuation costs, which we denote by  $\hat{W}$ 's, from actual continuation costs. Solving the model for this case is analogous, except that we proceed in two steps: (i) we solve for perceived future behavior and perceived continuation costs using beliefs  $\hat{\beta}$ ,  $\hat{\lambda}^Y$ , and  $\hat{\lambda}^N$  instead of  $\beta$ ,  $\lambda^Y$ , and  $\lambda^N$ ; and (ii) given the perceived continuation costs, we solve for actual behavior.

Step (i) is analogous to before, except that we use beliefs. In other words, we again use equations (1)–(5), except in all equations we replace  $\beta$ ,  $\lambda^Y$ ,  $\lambda^N$ ,  $\bar{c}_d$ ,  $W_d^Y$ ,  $W_d^N$ , and  $W_d$  with perceptions  $\hat{\beta}$ ,  $\hat{\lambda}^Y$ ,  $\hat{\lambda}^N$ ,  $\hat{c}_d$ ,  $\hat{W}_d^Y$ ,  $\hat{W}_d^N$ , and  $\hat{W}_d$ .

For step (ii), given perception  $\hat{W}_{d+1}$ , a person with  $s_d = Y$  will complete the task on day

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<sup>2</sup>The backward induction starts from equation (5), which can be combined with equations (2) and (1) to generate  $W_{D_N+1}$  and  $\bar{c}_{D_N}$ . Given  $W_{D_N+1}$  and  $\bar{c}_{D_N}$ , equations (1)–(4) can be used to generate  $W_{D_N}^Y$ ,  $W_{D_N}^N$ ,  $W_{D_N}$ , and  $\bar{c}_{D_N-1}$ . The recursion can then continue to generate the remaining  $\bar{c}_d$ 's.

$d$  when  $c_d + \beta\delta A_d \leq \beta\delta\hat{W}_{d+1}$ . Thus the actual cutoff cost is

$$\bar{c}_d = \beta\delta \left[ \hat{W}_{d+1} - A_d \right].$$

Hence, the probability that a person with  $s_d = Y$  will complete the task on day  $d$  is  $F(\bar{c}_d)$ . Finally, the unconditional probability of completing the task on day  $d$  is  $h_d \equiv \Lambda_d^Y F(\bar{c}_d)$ , where  $\Lambda_d^Y$  is defined by equation (6) (using the actual  $\lambda^Y$ ,  $\lambda^N$ , and  $\bar{c}_d$ 's because  $\Lambda_d^Y$  is tracking the actual proportion of the remaining population that has  $s_d = Y$ ).

### 3 Numerical Examples

We now present some numerical examples that shed insight on the identification of present bias and forgetting in this domain. For the examples in Figures 1-4 below, we consider a task with three deadlines ( $N = 3$ ), with  $D_1 = 20$ ,  $D_2 = 40$ ,  $D_3 = 60$ , and we assume  $\delta = 1$ . In the text, we consider two combinations of  $(a_1, a_2, a_3)$ . Additional examples appear in the Appendix; these consider an additional combination of  $(a_1, a_2, a_3)$ , and also consider an example with two deadlines ( $N = 2$ ) and an example with four deadlines ( $N = 4$ ). All examples assume endgame continuation costs  $z = 10$  and therefore  $Z = a_1 + a_2 + a_3 + 10$ .<sup>3</sup>

For the cost distribution  $F$ , the examples assume a simple two-parameter functional form  $F(c) = v + c/w$ , defined for  $c \in [0, (1 - v)w]$ . This functional form is convenient because the two parameters capture two key aspects of the cost distribution:  $v$  captures the mass at (or, more generally, near) zero, which has a major impact on the level of hazard rates, while  $w$  captures the spread of possible costs, which plays a major role in determining the magnitude of the slope leading up to a deadline.

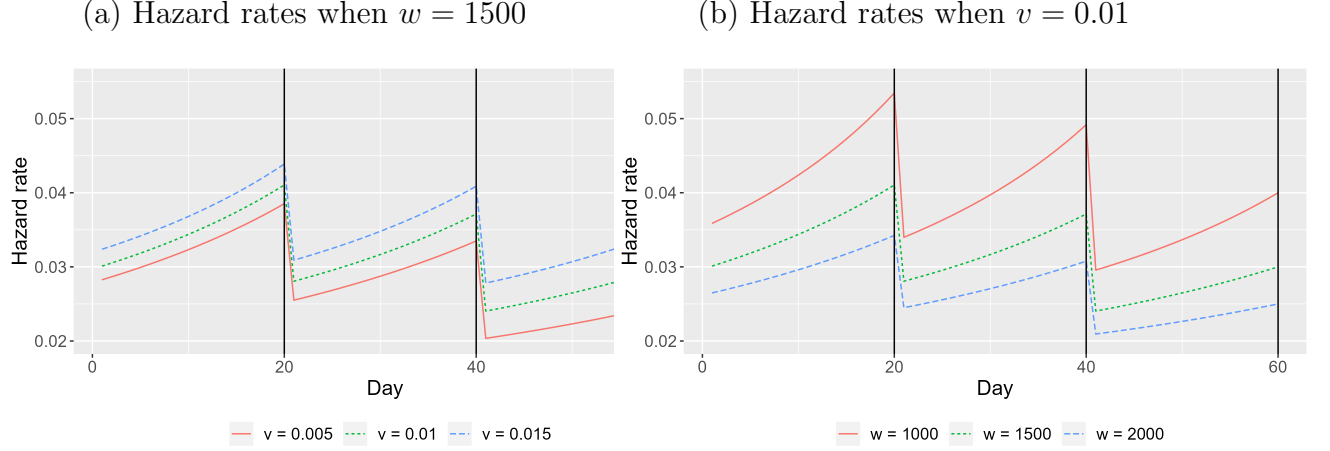
Importantly, by assuming a specific functional form for the cost distribution, we are in principle making identification *easier*. As we shall see, however, even when we assume this simple functional form, identification is problematic, and thus would be even more problematic without any functional-form assumptions.

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<sup>3</sup>Changes in  $z$  and changes in  $a_N$  have the same impact on behavior, and thus we do not consider different values of  $z$ .



Figure 1: Hazard Rates in Baseline Model



*Notes:* Both figures assume no present bias ( $\beta = \hat{\beta} = 1$ ) and no forgetting ( $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ); both figures assume  $\delta = 1$ ,  $z = 10$ , deadlines at days 20, 40, and 60, and penalties  $a_1 = a_2 = a_3 = 20$ ; and both figures assume cost distribution  $F(c) = v + c/w$ . Panel (a) reflects hazard rates when  $w = 1500$  for various values of  $v$ , while panel (b) reflects hazard rates when  $v = 0.01$  for various values of  $w$ .

### 3.1 Baseline model

Consider first a “Baseline” model in which there is no present bias ( $\beta = 1$ ) and no forgetting ( $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ). For this case, we can apply known results from similar optimal-stopping problems (e.g., Bertsekas (2005)) to conclude that, for any  $F$  (not just the functional form above), the effort-cost cutoffs satisfy  $\bar{c}_{d+1} \geq \bar{c}_d$  for all  $d \notin \{D_1, \dots, D_N\}$ , where the inequality is strict whenever  $F(\bar{c}_d) \in (0, 1)$ . Intuitively, the person faces a trade-off: she would like to complete the task before the next deadline to avoid the penalty, however, she would also like to find a convenient time. Well in advance of a deadline, it is safe to wait for a future low-cost day, but as that deadline approaches, the incentive to complete the task rises. Once that deadline passes, however, the incentive has a sudden drop, but then rises again toward the subsequent deadline. Figure 1 illustrates this behavior for several different combinations of  $v$  and  $w$  (the parameters of the cost distribution).

### 3.2 Identification of Present Bias

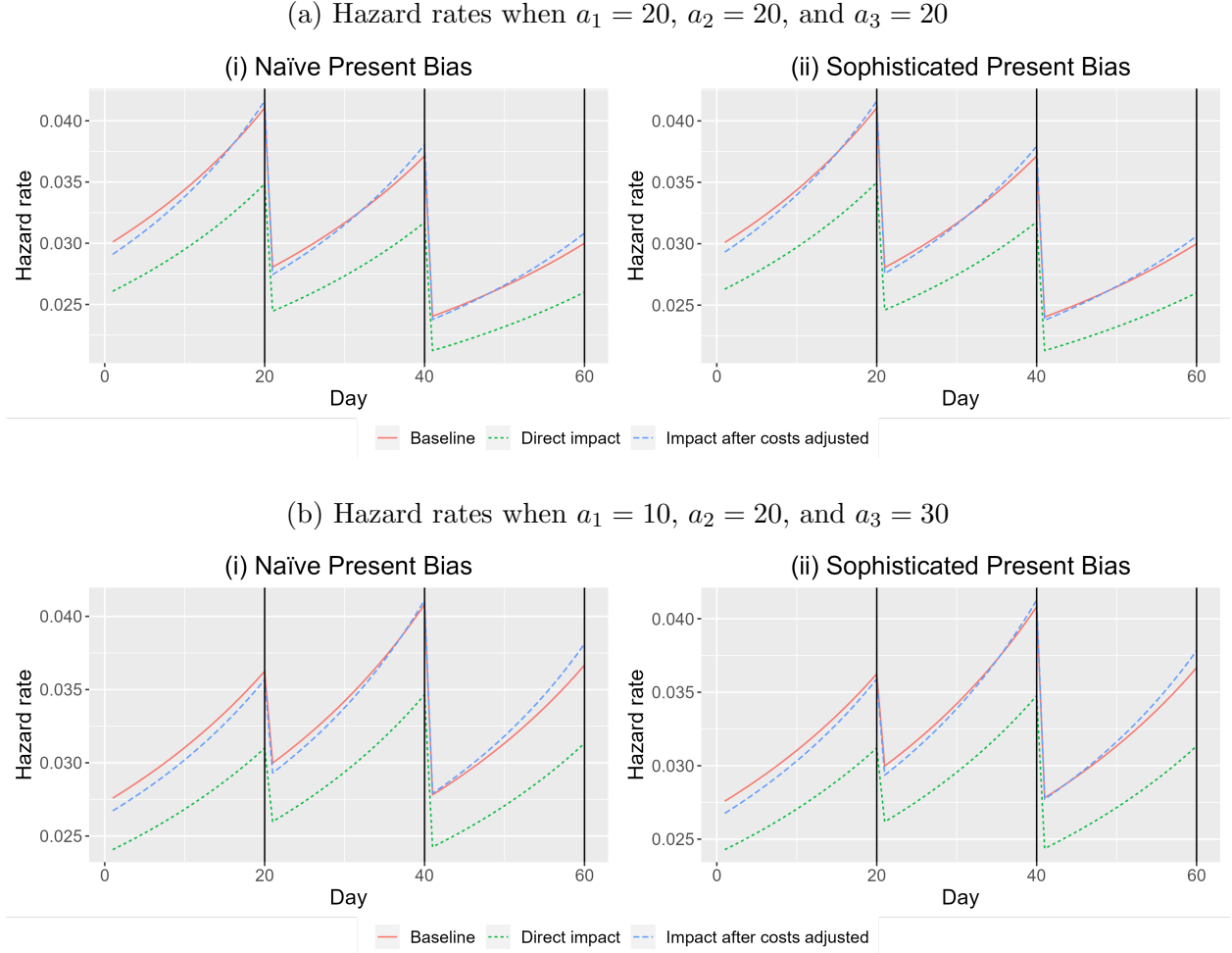
We now consider whether one can separately identify present bias and the parameters of the effort-cost distribution in this domain; for this analysis we assume no forgetting (i.e.,  $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ).

For this case, Heidhues and Strack (2021) prove for single-deadline tasks that, even if one could observe the stopping probabilities, one cannot identify present bias separately from the cost distribution. Their proof proceeds in two steps. First, they prove that, for single-deadline tasks, stopping probabilities for any  $\beta$  and  $\delta$  are (weakly) monotonically increasing up to the last day (the deadline). Second, they prove that, for any observed sequence of (weakly) increasing stopping probabilities, for any  $\beta$  and  $\delta$  one can construct a cost distribution that rationalizes that sequence. Their proof of the second step takes advantage of the first step: when constructing  $F$ , because the observed stopping probability increases from day  $d$  to day  $d + 1$ , one gets to construct  $F$  over a new interval in its support for which there are no restrictions.

Because for multiple-deadline tasks stopping probabilities are not (weakly) monotonically increasing up to the last day (as illustrated in Figure 1), the proof from Heidhues and Strack (2021) does not directly extend to this domain. However, we now illustrate via numerical examples that their result is in fact likely to extend.

In Figure 2, the solid red line labelled “Baseline” reflects hazard rates in the Baseline model when  $v = 0.01$  and  $w = 1500$ . Panels (ai) and (aii) illustrate the impact of instead assuming  $\beta = 0.8$  with (ai) naivete and (aii) sophistication when  $a_1 = a_2 = a_3 = 20$ . The short-dashed green line labelled “Direct impact” reflects hazard rates for  $\beta = 0.8$  while maintaining the same cost distribution as for the Baseline hazard rates. Ceteris paribus, the introduction of present bias leads to lower hazard rates, but does not change the qualitative pattern of hazard rates increasing up to each deadline and then dropping immediately after each deadline. Intuitively, present bias implies a person overly weights immediate effort costs and is thus less willing to act now, but the person is still reacting to deadlines. Interestingly, being aware of future present bias has very little impact on behavior—sophistication does have an impact, but it is sufficiently small that it is not visible in Figure 2.

Figure 2: Joint Identification of Present Bias and Cost Parameters



*Notes:* All four figures assume no forgetting ( $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ),  $\delta = 1$ ,  $z = 10$ , and deadlines at days 20, 40, and 60. Panels (a) and (b) differ in the penalties  $a_1$ ,  $a_2$ , and  $a_3$  for missing the three deadlines; within each panel, (i) assumes naivete ( $\hat{\beta} = 1$ ) while (ii) assumes sophistication ( $\hat{\beta} = \beta$ ). In each of the four figures, Baseline reflects hazard rates when  $\beta = 1$ ,  $v = 0.01$ , and  $w = 1500$ ; Direct impact reflects hazard rates when  $\beta = 0.8$ ,  $v = 0.01$ , and  $w = 1500$ ; and Impact after costs adjusted reflects hazard rates when  $\beta = 0.8$  and  $v$  and  $w$  are chosen to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. The chosen  $(v, w)$  are (ai) (0.009, 1100), (aii) (0.009, 1110), (bi) (0.011, 1180), and (bii) (0.010, 1150).

Although present bias does have a direct impact on behavior, separate identification of present bias ( $\beta$ ) and effort costs ( $F$ ) requires that a profile of observed hazard rates is consistent with only one combination of  $\beta$  and  $F$ . To highlight the identification challenge, the long-dashed blue line in panels (ai) and (aii) labelled “Impact after costs adjusted” presents hazard rates for  $\beta = 0.8$  after adjusting the parameters of the cost distribution ( $v$

and  $w$ ) to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. In other words, if  $h_d^{base}$  is the day- $d$  hazard rate under the Baseline model and  $h_d^{new}$  is the day- $d$  hazard rate under the model with present bias, we choose  $v$  and  $w$  to minimize<sup>4</sup>

$$\sum_{d=1}^{60} (h_d^{new} - h_d^{base})^2.$$

In both panels (ai) and (aia), we cannot perfectly replicate the solid red line. It follows that, with enough data, we could conclude that  $\beta = 1$  is separately identified from  $\beta = 0.8$ . In practice, however, the differences are so small that we could not reasonably identify  $\beta$  separately from the cost distribution in this domain. Moreover, even what little identification there is relies on assuming our specific two-parameter cost distribution. If, like Heidhues and Strack (2021), we did not impose any restrictions on  $F$ , we could make the long-dashed blue line even closer to the solid red line, and perhaps could even replicate it.

Panels (bi) and (bia) of Figure 2 perform an analogous analysis when  $a_1 = 10$ ,  $a_2 = 20$ , and  $a_3 = 30$ , Appendix Figure 5 panels (ai) and (aia) do so for  $a_1 = 30$ ,  $a_2 = 20$ , and  $a_3 = 10$ . For both cases, the message is much the same. The conclusion from Heidhues and Strack (2021) that present bias is not identified from task-completion data alone seems to extend—at least in practice—to multiple-deadline tasks.

### 3.3 Identification of Forgetting

While there is theoretical work applying forgetting to single-deadline tasks, there is no identification analysis analogous to what Heidhues and Strack (2021) do for present bias. There is also little work on forgetting in the context of multiple-deadline tasks. Here, we use a numerical analysis to shed some initial insight on whether one can separately identify forgetting and the parameters of the effort-cost distribution in this domain; for this analysis we assume no present bias (i.e.,  $\beta = 1$ ).

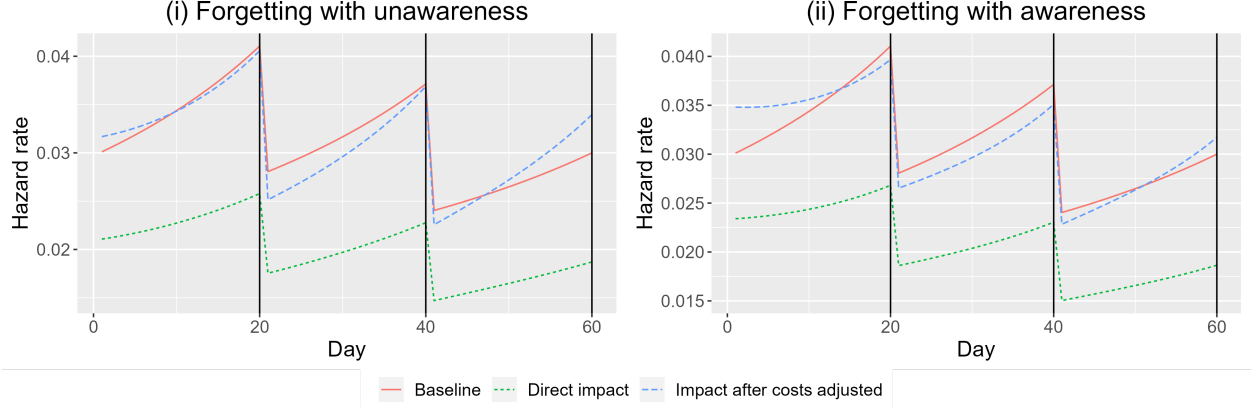
Panels (a) and (b) of Figure 3 are analogous to panels (a) and (b) of Figure 2, except they apply to the cases of forgetting with unawareness (ai and bi) and forgetting with awareness

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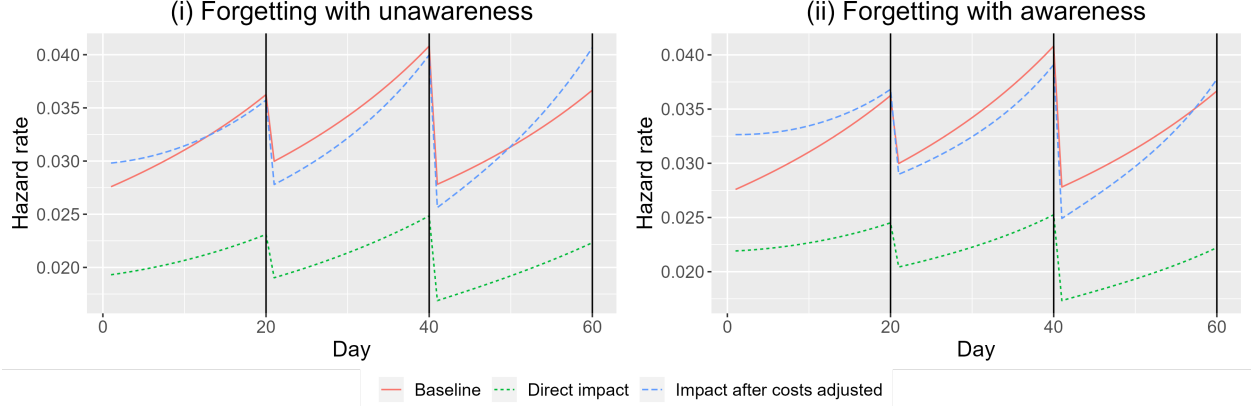
<sup>4</sup>We use a grid-search approach where the grid has a step size of .001 for  $v$  and 5 for  $w$ .

Figure 3: Joint Identification of Forgetting and Cost Parameters

(a) Hazard rates when  $a_1 = 20$ ,  $a_2 = 20$ , and  $a_3 = 20$



(b) Hazard rates when  $a_1 = 10$ ,  $a_2 = 20$ , and  $a_3 = 30$



*Notes:* All four figures assume no present bias ( $\beta = \hat{\beta} = 1$ ),  $\delta = 1$ ,  $z = 10$ , and deadlines at days 20, 40, and 60. Panels (a) and (b) differ in the penalties  $a_1$ ,  $a_2$ , and  $a_3$  for missing the three deadlines; within each panel, (i) assumes forgetting with unawareness ( $\hat{\lambda}^Y = \hat{\lambda}^N = 1$ ) while (ii) assumes forgetting with awareness ( $\hat{\lambda}^Y = \lambda^Y$  and  $\hat{\lambda}^N = \lambda^N$ ). In each of the four figures, Baseline reflects hazard rates when  $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ,  $v = 0.01$ , and  $w = 1500$ ; Direct impact reflects hazard rates when  $\lambda^Y = .98$ ,  $\lambda^N = .05$ ,  $\Lambda_1^Y = .7$ ,  $v = 0.01$ , and  $w = 1500$ ; and Impact after costs adjusted reflects hazard rates when  $\lambda^Y = .98$ ,  $\lambda^N = .05$ ,  $\Lambda_1^Y = .7$ , and  $v$  and  $w$  are chosen to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. The chosen  $(v, w)$  are (ai) (0.023, 735), (aii) (0.024, 860), (bi) (0.026, 720), and (bii) (0.019, 725).

(aai and bii). In each panel, the solid red line labelled “Baseline” reflects the same Baseline model used in Figure 2. The short-dashed green line labelled “Direct impact” reflects hazard rates for  $\lambda^Y = 0.98$ ,  $\lambda^N = 0.05$ , and  $\Lambda_1^Y = 0.7$  while maintaining the same cost distribution as for the Baseline hazard rates. Ceteris paribus, the introduction of forgetting leads to

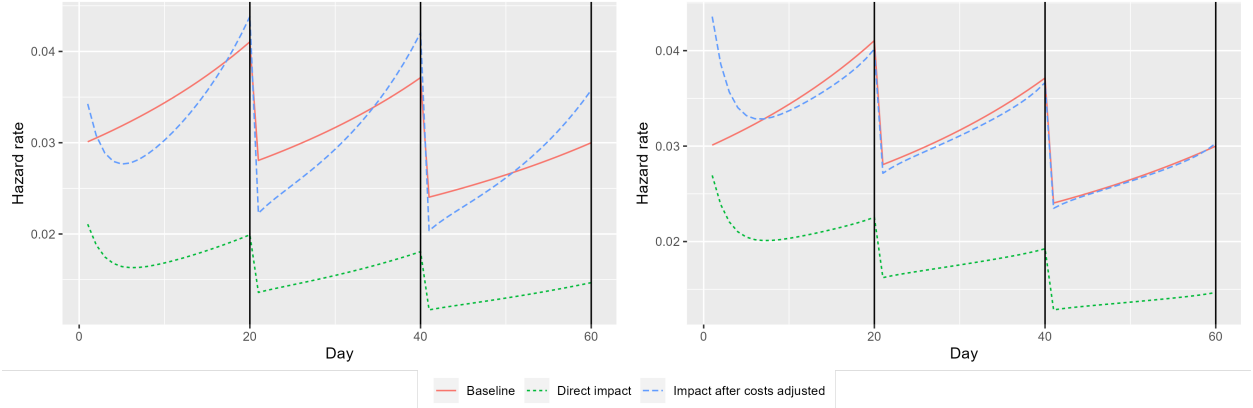
lower hazard rates, but does not change the qualitative pattern of hazard rates increasing up to each deadline and then dropping immediately after each deadline. Intuitively, forgetting means a person might not have the task on the mind and thus is less likely to act now, but anyone with the task on the mind is still reacting to deadlines. In addition, much like being aware of future present bias, being aware of future forgetting makes a person more likely to pay now, but also does not change qualitatively how a person reacts to deadlines.

Of course, separate identification of forgetting  $(\lambda^Y, \lambda^N, \Lambda_1^Y)$  and the effort-cost distribution  $(F)$  requires that a profile of observed hazard rates is consistent with only one combination of  $(\lambda^Y, \lambda^N, \Lambda_1^Y)$  and  $F$ . In each panel, the long-dashed blue line labelled “Impact after costs adjusted” presents hazard rates for  $\lambda^Y = 0.98$ ,  $\lambda^N = 0.05$ , and  $\Lambda_1^Y = 0.7$  after adjusting the parameters of the cost distribution ( $v$  and  $w$ ) to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. In general, we cannot perfectly replicate the solid red line, and there is perhaps more scope for identification of forgetting than for present bias. In practice, though, we suspect that even the more noticeable differences between the long-dashed blue line and the solid red line in Figure 3 would be too small to detect in any practical data set.

Figure 3 is perhaps misleading because we chose forgetting parameters that still generate the same pattern of hazard rates that we get from the Baseline model. If instead we chose parameters that permit more rapid forgetting ( $\lambda^Y$  enough smaller than one), there can be zones far from deadlines where hazard rates decline over time due to the task falling off people’s minds (as emphasized in Taubinsky (2014)). This pattern is illustrated in Figure 4, which uses  $\lambda^Y = 0.8$ ,  $\lambda^N = 0.2$ , and  $\Lambda_1^Y = 0.7$ .

If the underlying parameters were such that observed hazard rates exhibit zones of decreasing hazard rates, one could perhaps rely on such zones to identify the existence of forgetting. Here, however, one runs into the practical issue that we also need to deal with the possibility of unobserved heterogeneity, and it has long been emphasized how unobserved heterogeneity can generate zones of decreasing aggregate hazard rates. Hence, even when observed hazard rates exhibit such zones, we suspect it will still be difficult in practice to convincingly identify forgetting from task-completion data.

Figure 4: Joint Identification of Forgetting and Cost Parameters Revisited



*Notes:* Both figures assume no present bias ( $\beta = \hat{\beta} = 1$ ),  $\delta = 1$ ,  $z = 10$ , deadlines at days 20, 40, and 60, and  $a_1 = a_2 = a_3 = 20$ . Panel (i) assumes forgetting with unawareness ( $\hat{\lambda}^Y = \hat{\lambda}^N = 1$ ) while panel (ii) assumes forgetting with awareness ( $\hat{\lambda}^Y = \lambda^Y$  and  $\hat{\lambda}^N = \lambda^N$ ). In each of the two figures, Baseline reflects hazard rates when  $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ,  $v = 0.01$ , and  $w = 1500$ ; Direct impact reflects hazard rates when  $\lambda^Y = .8$ ,  $\lambda^N = .2$ ,  $\Lambda_1^Y = .7$ ,  $v = 0.01$ , and  $w = 1500$ ; and Impact after costs adjusted reflects hazard rates when  $\lambda^Y = .8$ ,  $\lambda^N = .2$ ,  $\Lambda_1^Y = .7$ , and  $v$  and  $w$  are chosen to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. The chosen  $(v, w)$  are (i)  $(0.005, 425)$  and (ii)  $(0.021, 705)$ .

## 4 Concluding Thoughts

Using numerical examples, we have demonstrated that identification of present bias is still problematic even when there are multiple deadlines, reinforcing the conclusion from Heidehues and Strack (2021). Identification of forgetting perhaps holds more promise, in part because forgetting can generate patterns that cannot emerge from a model without forgetting. Nonetheless, in practice, we suspect it too is problematic.

These conclusions perhaps should not be surprising. Typically, when economists look for sources of identification, we look for variation in the environment that primarily impacts behavior through a single mechanism. The premise of identifying present bias or forgetting from task-completion data alone is that one can treat distance to deadlines as the analogue of an exogenous shift to the environment. The problem is that the distance to the deadline also impacts hazard rates via the effort-cost distribution (and the incentive to delay to find a low-effort-cost day). Hence, variation in distance to deadlines is not a good means to distinguish present bias or forgetting from effort costs (or to distinguish present bias and

forgetting from each other, an issue that we have ignored).

A better approach is to combine task-completion data with other sources of (ideally exogenous) variation that operate more uniquely through one of the focal mechanisms. As one clear example, in Heffetz et al. (2022), there is an exogenous shift in the timing of reminder letters. Because such letters should only impact behavior if people have forgotten about a ticket, they represent a relatively clean way to identify the existence of forgetting.

Applying this logic to present bias, a natural approach is to look for task-completion data in which there is independent variation in effort costs versus monetary costs. A key marker of present bias is then that people should react asymmetrically to the two different types of costs (since one is immediate and the other is delayed). Unfortunately, because effort costs can be an internal (hard-to-observe) variable, such independent variation might be hard in practice to find.

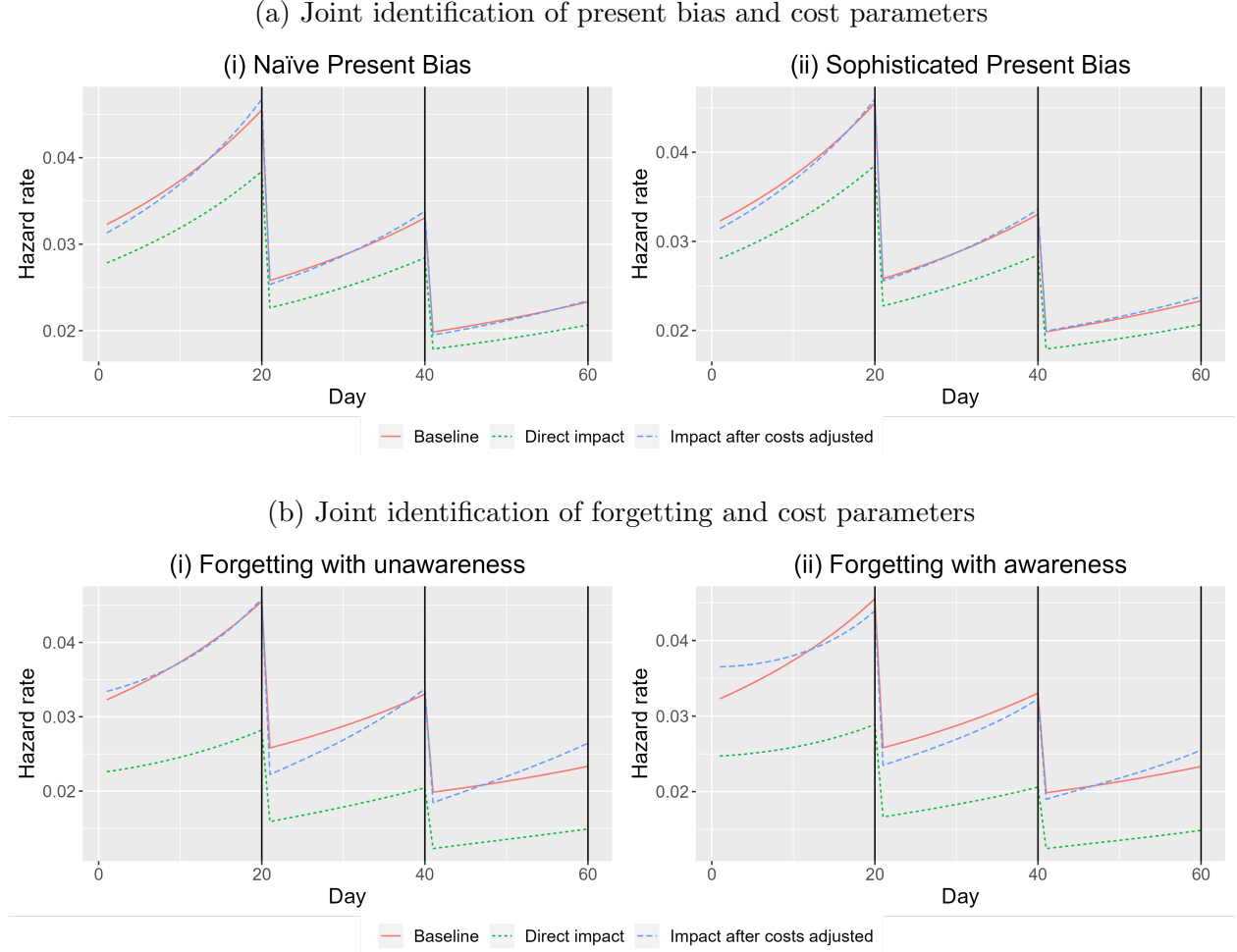


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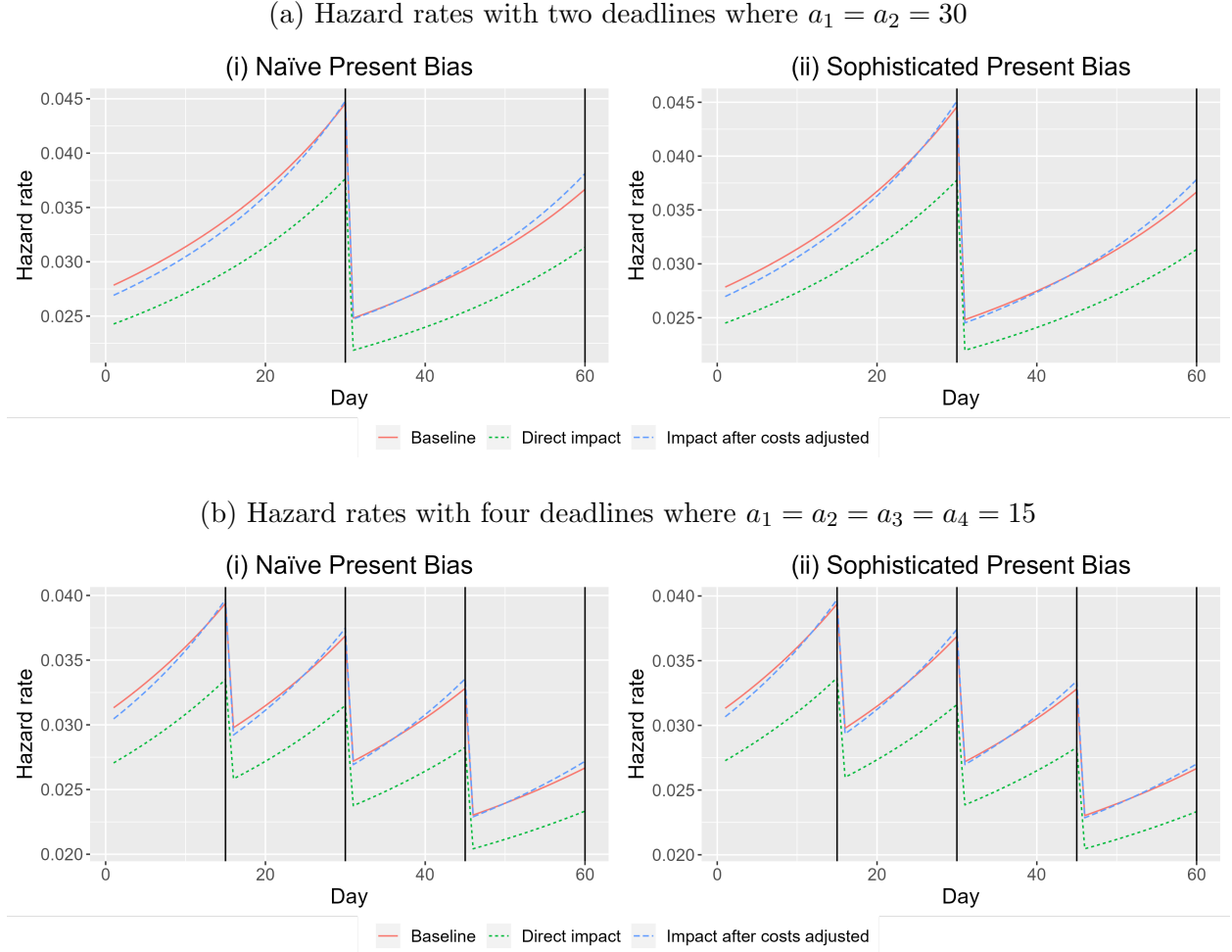
## Appendix: Additional Figures

Figure 5: Figures 2 and 3 with Declining Penalties ( $a_1 = 30$ ,  $a_2 = 20$ , and  $a_3 = 10$ )



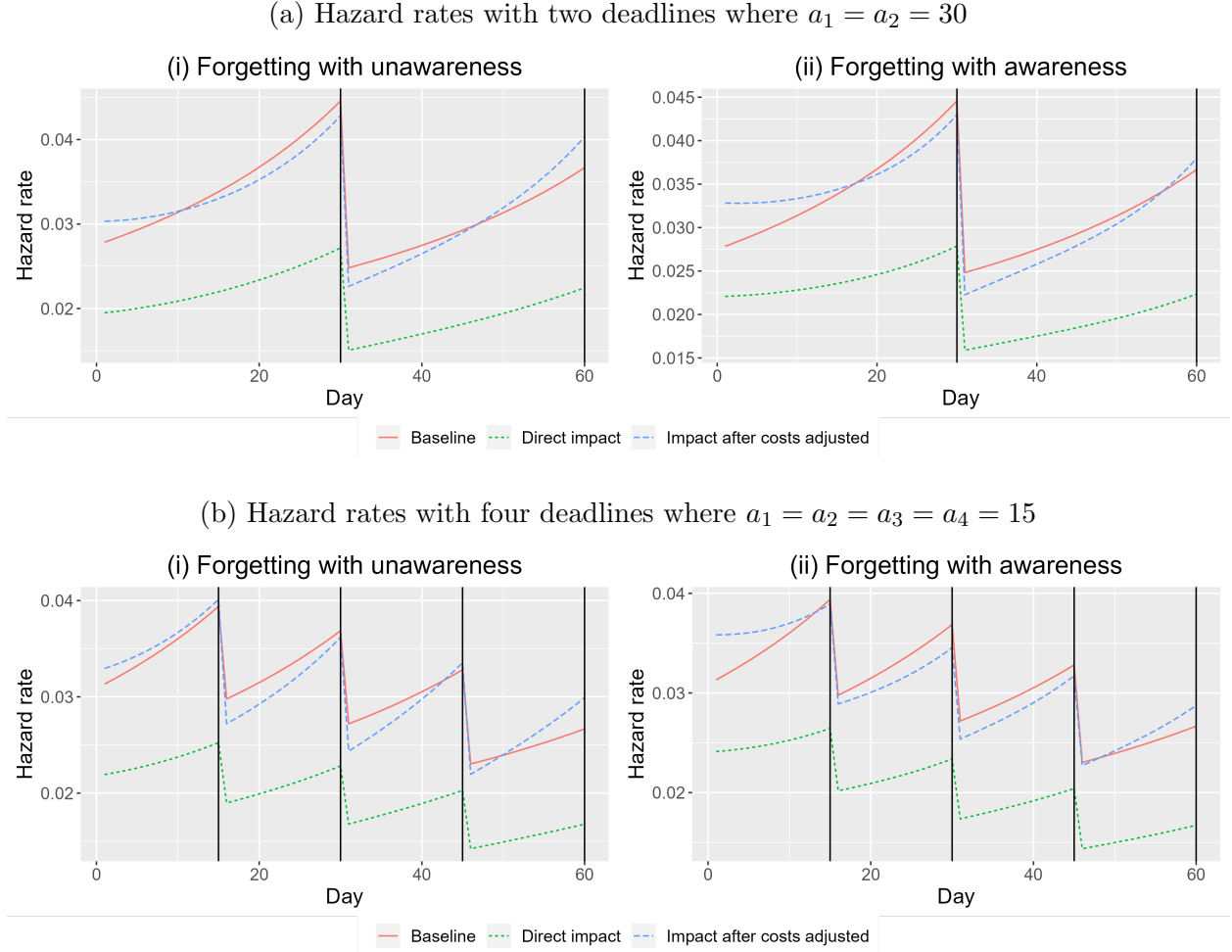
*Notes:* Except for assuming penalties  $a_1 = 30$ ,  $a_2 = 20$ , and  $a_3 = 10$ , panel (a) is otherwise identical to Figure 2, while panel (b) is otherwise identical to Figure 3. See notes from those figures for further details. Here, the chosen  $(v, w)$  for Impact after costs adjusted are (ai)  $(0.009, 1105)$ , (aii)  $(0.010, 1160)$ , (bi)  $(0.018, 700)$ , and (bii)  $(0.020, 810)$ .

Figure 6: Joint Identification of Present Bias and Costs with Two or Four Deadlines



*Notes:* All four figures assume no forgetting ( $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ),  $\delta = 1$ , and  $z = 10$ . Panel (a) assumes two deadlines, at days 30 and 60, with  $a_1 = a_2 = 30$ . Panel (b) assumes four deadlines, at days 15, 30, 45, and 60, with  $a_1 = a_2 = a_3 = a_4 = 15$ . Within each panel, (i) assumes naïveté ( $\hat{\beta} = 1$ ) while (ii) assumes sophistication ( $\hat{\beta} = \beta$ ). In each of the four figures, Baseline reflects hazard rates when  $\beta = 1$ ,  $v = 0.01$ , and  $w = 1500$ ; Direct impact reflects hazard rates when  $\beta = 0.8$ ,  $v = 0.01$ , and  $w = 1500$ ; and Impact after costs adjusted reflects hazard rates when  $\beta = 0.8$  and  $v$  and  $w$  are chosen to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. The chosen  $(v, w)$  are (ai) (0.011, 1180), (aii) (0.010, 1150), (bi) (0.009, 1100), and (bii) (0.009, 1110).

Figure 7: Joint Identification of Forgetting and Costs with Two or Four Deadlines



*Notes:* All four figures assume no present bias ( $\beta = \hat{\beta} = 1$ ),  $\delta = 1$ , and  $z = 10$ . Panel (a) assumes two deadlines, at days 30 and 60, with  $a_1 = a_2 = 30$ . Panel (b) assumes four deadlines, at days 15, 30, 45, and 60, with  $a_1 = a_2 = a_3 = a_4 = 15$ . Within each panel, (i) assumes forgetting with unawareness ( $\hat{\lambda}^Y = \hat{\lambda}^N = 1$ ) while (ii) assumes forgetting with awareness ( $\hat{\lambda}^Y = \lambda^Y$  and  $\hat{\lambda}^N = \lambda^N$ ). In each of the four figures, Baseline reflects hazard rates when  $\lambda^Y = \lambda^N = \Lambda_1^Y = 1$ ,  $v = 0.01$ , and  $w = 1500$ ; Direct impact reflects hazard rates when  $\lambda^Y = .98$ ,  $\lambda^N = .05$ ,  $\Lambda_1^Y = .7$ ,  $v = 0.01$ , and  $w = 1500$ ; and Impact after costs adjusted reflects hazard rates when  $\lambda^Y = .98$ ,  $\lambda^N = .05$ ,  $\Lambda_1^Y = .7$ , and  $v$  and  $w$  are chosen to minimize the sum of squared differences of daily hazard rates from Baseline hazard rates. The chosen  $(v, w)$  are (ai) (0.029, 790), (aii) (0.021, 760), (bi) (0.018, 680), and (bii) (0.024, 880).

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