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By

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Information Effects of Jump Bidding in English Auctions

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Abstract

Should an auctioneer start a rising auction from some starting price or set it as a reservation price? Under what circumstances might a bidder find it rational to raise the current offer by a substantial factor instead of making just a small increase above the highest bid? This paper aims to answer both of these questions by exploring the implications of jump bidding over the information sets available to the bidders. Our motivation is to find whether hiding the information about other players' signals might be beneficial for one of the bidders. We first show that it is better for the auctioneer to set a reservation price rather than "jump" to the starting price. We then prove that in a very general setting and when bidders are risk-neutral there exist no equilibrium with jump bidding (in non-weakly dominated strategies). Finally, we demonstrate that jump bidding might be a rational consequence of risk aversion, and analyze the different effects at work.

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1 Introduction

For years now the field of auctions has served as an unlimited source for behavioral phenomena which seemingly contradicted the theoretical predictions. The current paper focuses on a specific type of behavior which was observed both in controlled test environments and in real-life auctions, namely jump bidding in English auctions. A jump bid consists of raising the bid by an increment larger than the minimal required. Considering the standard model of English auctions, this seems to be quite an odd behavior. When jump bidding, a bidder might bid above the other bidders' valuations and pay more than she would have, had she not jumped. Moreover, if the jump does not pass the others' valuations, there is no reason to suspect they would change their behavior and drop out before they reach their valuation. While peculiar from a game-theoretic point of view, this behavior appears to be quite widespread. Isaac, Salmon, and Zillante (2007) have gathered data from over 40 auctions performed in the United States and the UK, and show that most of them contain jump bids of different sorts. The jump bids occur both at the beginning of the auction and during the bidding process. Some of the jump bids are by a relatively small fraction of the final price, while others are much more substantial. More indications that jump bids are common are provided in McAfee and McMillan (1996), Daniel and Hirshleifer (1997), Wang and Gunderson (1998) and Avery (1998).

Most of the jump bidding literature is concerned with proving existence and non-existence of equilibria that contain jump bids. Related papers which prove non-existence assume the auctioneer has the right to reject bids (Dodonova and Khoroshilov, 2006), or independent values and separable signals support for one of the bidders (Wang and Gunderson, 1998). Not surprisingly, while most non-existence results in the literature are quite robust, the existence results are loosely based on rather implausible assumptions about the auction design or the bidders behavior. Avery (1998) suggests that jump bidding can be used to signal one has a high valuation for the auctioned item. While this approach indeed explains collusion well in repeated auctions, Avery's equilibrium is drawing on weakly-dominated strategies (the other player does not continue bidding after the jump, even if her expected value exceeds the jump bid) and thus is not convincing when considering "one-shot" auctions. Other researchers try to add an external cost mechanism such as time-dependent utility (Kwasnica and Katok, 2005), transaction costs (Easley and Tenorio, 2001), research costs (Fishman, 1988; Hirshleifer and Png, 1989), and costs incurred by auctioning multiple objects (Zheng, 2008). Finally there have also been some attempts to explain jump bidding by assuming some discontinuity in the auction design (Isaac, Salmon, and Zillante, 2007), or in the bidders' preferences (Wang and Gunderson, 1998).

Our approach is largely motivated by the results of Milgrom and Weber (1982) which show that the information provided by the losing bidders in an English auction, as opposed to a second-price sealed-bid auction, causes a raise in the expected final price paid by the winner. This result obtains due to the information passed by the "low-signaled" bidders to the "high-signaled" bidders, hence the price-setters. Therefore, disallowing the "low-signaled" bidders to reveal their values might have the opposite effect. Obviously, this idea can only be applied in an interdependent values environment, in order for this information to have any significance for the bidders. We use similar arguments to show that an auctioneer is better off setting a reservation price, rather than starting the auction from this price.

Even though this result motivates the research of information hiding as a strong incentive for jump bidding, we continue by showing non-existence of jump bidding equilibria in a symmetric interdependent values and affiliated signals environment with risk-neutral bidders. For this purpose the deletion of weakly-dominated strategies suffices, and is much more economical than the assumptions made in previous studies. Moreover, our approach demonstrates intrinsic qualities of the bidders' strategies and reveals that the main reason for non-existence is the information revealed by the jump bid. We proceed by removing some of our earlier restrictions, and show that when bidders are risk-averse there are two good reasons why they should jump bid: to induce lotteries on other players and to avoid lotteries for themselves. The analysis provides both with tools to evaluate the different effects of information hiding, as well as a simple and reasonable intuition as to what might drive bidders to jump bid.

The rest of the paper is organized as follows. Section 2 contains a model of English auction, which also permits jump bidding. We continue in Section 3 by providing the initial intuition for information hiding, and comparing starting prices with reservation prices. An inexistence result for the case of risk-neutrality is then presented in Section 4, and we proceed in Section 5 by demonstrating how the introduction of risk-averison affects the jump bidding decision. Section 6 concludes.

2 The Model

We use the classical framework of English auction with interdependent-values and affiliated signals, as described by Milgrom and Weber (1982):

> [...] the price is posted using an electronic display. The price is raised continuously, and a bidder who wishes to be active at the current price depresses a button. When he releases the button, he has withdrawn from the auction.[1]

Formally, N bidders compete for a single indivisible object, auctioned by the auctioneer. The (von Neumann-Morgenstern) utility of the auctioneer is defined by:

$$u_s = \begin{cases} p & \text{if object is sold} \\ s & \text{if object is not sold} \end{cases}$$

where p is the price in which the object was sold, and S is the value of the object for the auctioneer. Each bidder receives a private signal x_i, where the signals are drawn from a given joint cumulative distribution function F, with the respective joint density function f. Define $v_i : R^N \to R$, the value of the object for player i as a function of the N signals. Define

$$u_i = \begin{cases} v_i - s_i & \text{if } s_i > \max_{j \neq i} \{s_j\} \\ 0 & \text{else} \end{cases} \tag{2.1}$$

a quasi-linear utility function, where s_j is the last bid of player j. It is assumed that v_i is non-negative, continuous and non-decreasing in all its parameters. Also, for each i, v_i satisfies the single crossing condition: $\forall j \neq i$: $\frac{\partial v_i}{\partial x_i} > \frac{\partial v_i}{\partial x_j}$. It is further assumed that the signals induced by F are affiliated, i.e.: let z and z' be points in R^N, let $z \vee z'$ denote the component-wise maximum of z and z', and let $z \wedge z'$ denote the component-wise minimum, then: $f(z \vee z')f(z \wedge z') \geq f(z)f(z')$. The environment is assumed to be symmetric, meaning that for each i: $v_i = v(x_i, x_{-i})$, the function v is symmetric with respect to components of the vector x_{-i}, and f is symmetric with respects to all its N arguments. Finally, We relax the model's assumptions by allowing

[1]Milgrom and Weber (1982, page 1104). This model is based on a description in: Cassady R. Jr., Auctions and Auctioneering, Berkeley, University of California Press, 1967.

jump bid upon the first bid, either by the auctioneer (Section 3) or by one of the bidders (Section 4).

Throughout this paper it will be convenient to denote by $w_{i:j}$ the vector $(w_i, w_{i+1}, \ldots, w_{j-1}, w_j)$ for any vector w and every $i \leq j$.

3 Starting Prices vs. Reservation Prices

Definition 3.1. A **"Cut-T auction"** is an English auction which starts with the value of T, i.e. the lowest bid possible is T. Following the English auction's description quoted above, in a Cut-T auction the electronic display will begin at T. Obviously, a Cut-T auction with $T = 0$ is equivalent to a standard English auction.

In order to describe an equilibrium in a Cut-T auction, we shall now explicitly define what a player's strategy consists of. A strategy in a Cut-T auction is a pair $\left(y^T, \{b_m\}_{m=1}^N\right)$. The first element is a cut-off point, and defines the lowest signal that the player must receive in order to enter the auction. The second element is a set of functions, which will be used by the player in the case she enters the auction. Each function corresponds to the m bidders which are currently active in the auction. Each b_m is a function from the player's signal and the entire history of the auction to the player's drop-out price.

Lemma 3.1. *In a symmetric equilibrium of a Cut-T auction, which consists of strictly monotonically rising (in the players' respective signals) bidding functions, y^T is defined uniquely by the implicit expression:*

$$E\left(v_j | X_j = y^T, X_j \geq \max_k \{X_k\}\right) - T = 0 \tag{3.1}$$

Proof. See Appendix. □

Once a bidder decides to participate in the auction, a symmetric equilibrium strategy must resemble the strategies of the English auction symmetric equilibrium as constructed in Milgrom and Weber (1982). In an English auction the bidders are able to deduce the other bidders' signals from their drop-out prices. Milgrom and Webber show that the unique symmetric equilibrium is defined iteratively by the following strategies (the strategies written here are for player 1):

$$b_N(x_1) = E\left(v_1 \,|\forall 1 \leq i \leq N : X_i = x_1\right)$$

$$b_k\left(x_1|p_{k+1},\ldots,p_N\right)=E\left(v_1\left|\begin{array}{l}\forall 1\le i\le k: X_i=x_1,\\ \forall k<i\le N: b_i\left(X_i|p_{i+1},\ldots,p_N\right)=p_i\end{array}\right.\right)$$

Where p_i is the drop-out price of player i, and the buyers are sorted according to their signals. k represents the number of buyers still active in the auction. Note that using these strategies, the drop out price of player 2 (and therefore the final price) is:

$$E\left(v_2\left|X_1=x_2,X_2=x_2,b_3\left(X_3|p_4,\ldots,p_N\right)=p_3,\ldots,b_N(X_N)=p_N\right.\right)$$

In the Cut-T auction the equilibrium strategies are almost identical but since the buyers that decide not to participate do not reveal their signals, the participating buyers will only be able to use the information that their signal level is below y^T. Therefore, in a Cut-T auction where only the first m bidders decided to participate, the strategies for player 1 are defined iteratively by:

$$b_m(x_1)=E\left(v_1\left|\forall 1\le i\le m: X_i=x_1,\forall m<i\le N: X_i<y^T\right.\right)$$

$$b_k\left(x_1|p1,\ldots,p_k\right)=E\left(v_1\left|\begin{array}{l}\forall 1\le i\le k: X_i=x_1\\ \forall k<i\le m: b_i\left(X_i|p_{i+1},\ldots,p_m\right)=p_i\\ \forall m<i\le N: X_i<y^T\end{array}\right.\right)$$

Proposition 3.1. *Considering the symmetric equilibrium with strictly monotonically increasing strategies, and $T=S$ (S is the object value for the auctioneer) the expected revenue from a Cut-T auction is lower than the revenue from a Standard English auction with a concealed reservation price of T.*

Proof. See Appendix. □

Put in simple words, Milgrom and Weber (1982) prove that revealing information raises the price, and the above proof relates this to the observation that starting the auction at price T obscures the information provided by the low-bidders' signals (which are affiliated with the high-bidders' signals). Proposition 3.1 can be applied to the scenario in which an auctioneer would like to auction an object and set a reservation price which equals to her private utility from the object.[2] It implies that even though the reservation price is known in advance, it is still better to let the buyers start from

[2]It appears that setting a starting price instead of using a reservation price is quite common in some online auctions sites, such as eBay (However, eBay's online guide specifically warns that setting the starting price too high might discourage potential bidders).

zero, thus revealing their private information. Note that in order for the low-bidders to have an incentive to play the equilibrium strategies it is important for the reservation price to be concealed. Nevertheless, even if the reservation price is known and some of the low-bidders play a different strategy, still as long as information can be extracted from their actions it is better for the auctioneer to select an English auction with a known reservation price over a Cut-T auction.

4 Inexistence of Jump Bidding Equilibria

Section 3 reinforces our basic intuition that by hiding information, starting at a higher price can reduce the revenue for the auctioneer. If so, every bidder would prefer (ex-ante) to participate in a Cut-T auction rather than a standard English auction. The question remains whether a bidder can mimic this result by independently applying a jump bid at the beginning of an English auction. In order to explore the ramifications of a jump bid made by the bidder we alter the basic model to allow a jump bid by a single player. We distinguish player 1 from the rest by allowing her, upon the first move, to jump to any positive bid. If a jump occurred then every player (other than player 1) decides whether or not to continue. If no player continues then player 1 wins the object and pays her bid.

In what follows we show that it is not profitable for player 1 to jump. The idea behind the proofs is that the player with the highest signal besides player 1 will always be able to deduce the correct probability distribution for the signals of the players which were "jumped upon", and moreover, will be able to use the information implied in the jump bid to her advantage. Proposition 4.1 handles the easier case of a strictly monotonically increasing jumping strategy and the result is later extended through proposition 4.2 to weakly increasing jumping strategy.

Definition 4.1. A **natural equilibrium** is a Bayesian equilibrium in non-weakly dominated strategies in which players 2 to N play a symmetric and strictly monotonically increasing (in their respective signals) strategies.

Proposition 4.1. *There is no natural equilibrium in which player 1 uses a jumping strategy which is* ***strictly*** *monotonically increasing in her signal.*

Proof. The intuition that guides the proof is quite simple. When player 1 jumps she reveals her signal, and since information hiding is only useful to obscure the highest signal, it becomes redundant and in some cases even harmful for her to jump. For the complete proof see Appendix.

□

Lemma 4.1. *There is no natural equilibrium in which player 1 uses a weakly monotonically increasing (in her signal) jumping strategy* $s : R \to R$*, and there exists* $0 < t \in R$*, such that* $s^{-1}(t)$ *has a least element.*

Proof. The proof relies on the observation that for a bidder who receives the signal $\min\{s^{-1}(t)\}$, jumping is strictly dominated. For the complete proof see Appendix. □

Proposition 4.2. *If the probability distribution* F *has no singleton atoms, then there is no natural equilibrium in which player 1 uses a non-zero weakly monotonically increasing (in her signal) jumping strategy.*

Proof. See Appendix. □

5 The Effects of Risk Aversion on the Jump Bidding Decision

Our comparison of starting prices and reservation prices in section 3 had focused on the effect caused by the affiliation between the players' signals, which makes the second-highest bidder's expected valuation lower when the information provided by the affiliated variables remained unobserved. The current section deals with a completely independent outcome of information hiding, namely the risk implications of the jump. We shall now drop our earlier assumption of risk-neutrality, and demonstrate that when bidders are risk-averse, jump bidding can indeed be profitable. The intuition gained in our previous analysis should give us insight as to what changes might be needed in order for the jump bid to be successful. Our first two examples might seem somewhat peculiar, as two of the bidders have an extremely low valuation of the bidded object, and their actions follow only from the assumption of using non-weakly dominated strategies. Moreover, for the sake of simplicity and to illustrate the independence between the current results and the previous, we shall even drop the affiliation between the jumper and the rest of the players.

Example 5.1 (Risk-averse second-highest bidder)**.** *Let X_3 be either 0 or 1 with equal probabilities. The rest of the signals bear no information, $X_1 = X_2 = 0$. The values are given by:*

$$\begin{aligned} v_1(X_1, X_2, X_3) &= 1000 \\ v_2(X_1, X_2, X_3) &= 100 + 10X_3 \\ v_3(X_1, X_2, X_3) &= X_3 \end{aligned}$$

bidders 1 and 3 are risk-neutral, while bidder 2 (the second-highest bidder) is risk-averse:

$$u_2 = \begin{cases} u(v_2 - s_2) & \textit{if } s_2 > \max_{j \neq 2} \{s_j\} \\ u(0) & \textit{else} \end{cases} \tag{5.1}$$

where u satisfies $u' > 0$ and $u'' < 0$. Note that the non-weakly dominated strategies are fairly simple: bidder 1 stays until the price-clock reaches 1000, bidder 3 stays until the price-clock reaches X_3. If bidder 2 does not know X_3 she stays only until the price-clock reaches s, where s is implicitly defined by:

$$\frac{1}{2} \cdot (u(110 - s)) + \frac{1}{2} \cdot (u(100 - s)) = u(0)$$

But as u is convex we get:

$$u(105 - s) = u(\frac{1}{2} \cdot (110 - s) + \frac{1}{2} \cdot (100 - s)) > u(0)$$

And so $s < 105$. Now note that:

$$U_1^{\textit{No-Jump}} = \frac{1}{2} \cdot (1000 - 110) + \frac{1}{2} \cdot (1000 - 110) = 1000 - 105 < 1000 - s = E_1^{\textit{Jump}}$$

Bidder 1 is better off if she jumps above bidder 3. The effect stems from the fact bidder 2 prefers the expected utility over the utility from the lottery she is facing when the signal of bidder 3 is unknown.

Our first example was fairly intuitive and suggested that if the first bidder is risk-neutral and knows that another bidder is risk-averse, she should exploit this to get a reduced price by exposing the other bidder to the lottery induced by the jump bid. The next example is somewhat more surprising as it claims that a risk-averse bidder who knows that other bidders are risk-neutral should also jump and hide information.

Example 5.2 (Risk-averse jumper)**.** *Let X_i and v_i be as before, but now assume bidders 2 and 3 are risk-neutral and it is bidder 1 (the jumper) who is risk-averse (using the same form of utility as in* (5.1)*). Again, bidder 1 stays until the price-clock reaches 1000, bidder 3 stays until the price-clock reaches X_3, and bidder 2 stays until the price-clock reaches* $100 + 10X_3$ *if she deduced X_3 from bidder 3 actions, or until the price-clock reaches* $\mathrm{E}\left(v_2(X_1, X_2, X_3)\right) = 105$ *if she did not deduce X_3. Considering the strategy of no-jump by bidder 1 versus the strategy of jumping the price* $1 + \epsilon$ *we get:*

$$U_1^{\textit{No-Jump}} = \frac{1}{2} \cdot (u(1000 - 110)) + \frac{1}{2} \cdot (u(1000 - 110)) <$$
$$u\left(\frac{1}{2} \cdot (1000 - 110) + \frac{1}{2} \cdot (1000 - 100)\right) = u(1000 - 105) = U_1^{\textit{Jump}}$$

And that means bidder 1 is better off if she jumps above the low-valued player. Intuitively, bidder 1 is risk-averse and so she prefers the certainty of the expected value over the lottery caused by the price uncertainty induced by bidder 3's signal.

It is important to spell out the difference between these two examples and the former discussed aspect of information hiding. The propositions in section 4 prove that a bidder who tries to hide the affiliated signals is doing so in vain, as her signal (or at least, its affiliated properties) will be revealed through the jump bid. Even breaking the symmetry and separating the bidders' valuations (as we did in the above examples) could not have made the jump bidding profitable in the case of risk-neutrality, due to the same argument. However, risk aversion does not work that way and the motivation for the jump is practically uncorrelated with the jumper's signal. The jumper either tries to avoid the "lottery" induced by the (unobserved) signals of bidders with low-valuations or to conceal the results of the same lottery from other risk-averse bidders in order to lower their expected utility from winning the auction.

Let us consider a more general and symmetric model and observe the different effects at work. Assume that there are m "high" bidders and $(N-m)$

"low" bidders whose utilities are given by:[3]

$$\forall 1 \le i \le m : u_i = \begin{cases} u(v_H - s_i) & \text{if } s_i > \max_{j \neq i} \{s_j\} \\ u(0) & \text{else} \end{cases}$$

$$\forall m < i \le N : u_i = \begin{cases} u(v_L - s_i) & \text{if } s_i > \max_{j \neq i} \{s_j\} \\ u(0) & \text{else} \end{cases}$$

Where $v_H : R^N \to R$ and $v_L : R^N \to R$ are functions from the players' signals (which are again drawn from a joint distribution F, which should be symmetric only with regard to the "high" bidders). v_H is symmetric with regard to the other "high" bidders' signals, and to simplify notation, also with regard to the other "low" bidders' signals (as in section 2), and both value functions are "separated" such that:

$$\inf \{supp(v_H)\} > \sup \{supp(v_L)\}$$

We shall now try to find when does a strategy which always starts with a jump to $\sup \{supp(v_L)\} + \epsilon$ dominate the strategy which never start with a jump (the deterministic property of the jump is to eliminate the possibility that the other players learn anything from the jump). Given a certain vector of signals $x_{2:m}$ for $(m-1)$ high players, let us denote by $\bar{p}(x_{2:m})$ the price that the player with the highest signal of those (we shall assume throughout that players $2, \ldots, m$ are sorted by their signals) would be willing to pay in equilibrium after a jump was performed. $\bar{p}(x_{2:m})$ is given implicitly by the formula:[4]

$$\underset{X_{1:N}}{\mathrm{E}} \left(u \left[v\left(X_{1:N}\right) - \bar{p}\left(X_{2:m}\right) \right] | X_1 = x_2, X_{2:m} = x_{2:m} \right) = 0$$

Now let us define three expressions that represent the revenues gained from information hiding through jump bidding:

Inducing lotteries effect - creating a value-lottery for player 2

$$A(x_1) \equiv \underset{X_{1:N}}{\mathrm{E}} \left(u \left[v(X_{1:N}) - \bar{p}(X_{2:m}) \right] | X_1 = x_1, X_1 > X_2 \right) -$$
$$\underset{X_{1:N}}{\mathrm{E}} \left(u \left[v(X_{1:N}) - \underset{Y_{1:N}}{\mathrm{E}} \left(v(Y_{1:N}) \middle| \begin{array}{l} Y_1 = X_2, \\ Y_{2:m} = X_{2:m} \end{array} \right) \right] \middle| \begin{array}{l} X_1 = x_1, \\ X_1 > X_2 \end{array} \right)$$

[3]Note that the risk-aversion property of the low bidders is immaterial to the analysis, and is only used to make the separation property simpler.

[4]In what follows we shall assume that every expectation operator is also conditional on the signals of players $3, \ldots, m$ being sorted, and so are the signals of players $m+1, \ldots, N$.

Avoiding lotteries effect - eliminating a price-lottery for player 1

$$B(x_1) \equiv \mathop{\mathrm{E}}_{X_{1:N}} \left(u \left[v(X_{1:N}) - \mathop{\mathrm{E}}_{Y_{1:N}} \left(v(Y_{1:N}) \middle| \begin{array}{l} Y_1 = X_2, \\ Y_{2:m} = X_{2:m} \end{array} \right) \right] \middle| \begin{array}{l} X_1 = x_1, \\ X_1 > X_2 \end{array} \right) -$$
$$\mathop{\mathrm{E}}_{X_{1:N}} \left(\mathop{\mathrm{E}}_{Y_{1:N}} \left(u \left[v(X_{1:N}) - v(Y_{1:N}) \right] \middle| \begin{array}{l} Y_1 = X_2, \\ Y_{2:m} = X_{2:m} \end{array} \right) \middle| \begin{array}{l} X_1 = x_1, \\ X_1 > X_2 \end{array} \right)$$

Hidden affiliation effect - Disrupting information flow to player 2

$$C(x_1) \equiv \mathop{\mathrm{E}}_{X_{1:N}} \left(\mathop{\mathrm{E}}_{Y_{1:N}} \left(u \left[v(X_{1:N}) - v(Y_{1:N}) \right] \middle| \begin{array}{l} Y_1 = X_2, \\ Y_{2:m} = X_{2:m} \end{array} \right) \middle| \begin{array}{l} X_1 = x_1, \\ X_1 > X_2 \end{array} \right) -$$
$$\mathop{\mathrm{E}}_{X_{1:N}} \left(\mathop{\mathrm{E}}_{Y_{1:N}} \left(u \left[v(X_{1:N}) - v(Y_{1:N}) \right] | Y_{1:m} = X_{1:m} \right) | X_1 = x_1, X_1 > X_2 \right)$$

The next expression we introduce represents the lose that a bidder might encounter due to a jump bid:

Values correlation effect - Breaking the connection between the first bidder's gains and losses

$$D(x_1) \equiv \mathop{\mathrm{E}}_{X_{1:N}} \left(u \left[v(X_{1:N}) - v(X_{1:N}) \right] \middle| X_1 = x_1, X_1 > X_2 \right) -$$
$$\mathop{\mathrm{E}}_{X_{1:N}} \left(\mathop{\mathrm{E}}_{Y_{1:N}} \left(u \left[v(X_{1:N}) - v(Y_{1:N}) \right] | Y_{1:m} = X_{1:m} \right) \middle| X_1 = x_1, X_1 > X_2 \right)$$

Observation 5.1. *$A(x_1)$, $B(x_1)$, and $C(x_1)$ are non-negative for any x_1, and jumping over the low bidders dominates non-jumping iff:*

$$\forall x_1 : A(x_1) + B(x_1) + C(x_1) \geq D(x_1)$$

Proof. See Appendix. □

The meaning of observation 5.1 is that in a general environment, the first bidder (as are all other bidders) benefits from jump bidding in three different ways. First, the risk aversion of the other bidders causes a decrease in the expected price. Second, the bidder enjoys a more "stable" result, as there is no perceived variation in the lower signals. Third, the hiding of the affiliated signals causes yet another decrease in the final auction price. It is important to note the possible "counter-effect" which appears in observation 5.1. This effect is the reason we cannot provide a "clean-cut" result. As the first bidder cannot consider her value to be uncorrelated with the others' values, the differentiation between the two expectations might lower the expected revenue from jumping. It is quite straightforward that for low values of $X_{(m+1):N}$ the expected value of $v(X_{1:m}, Y_{(m+1):N})$ will be higher than $v(X_{1:N})$, while the opposite holds for high values of $X_{(m+1):N}$. The expected value over all the possible values cannot be predicted without making further assumptions on the behavior of u and v. Finally, similarly to the two preceding examples, it is the separation property that allows the first bidder to jump without revealing anything about her private value. When bidders are risk-neutral we get $A(x_1) = B(x_1) = C(x_1) = D(x_1) = 0$, and this shows that even when separation is possible, affiliation by itself does not provide enough motives for a bidder to jump.

6 Discussion

In this paper we have examined the possibility of explaining the jump bidding phenomenon as a rational attempt made by a player to lower the expected final price of the object through information hiding. The motivation arose from the important result by Milgrom and Weber (1982), showing that generally revealing information raises the expected revenue for the auctioneer, hence the payment made by the winning bidder, and a jump bid is one way the players might prevent the spreading of information. By comparing starting prices to reservation prices we have shown that the basic idea of information hiding indeed works for the benefit of the bidders. However, we have also proved that such course of action, when attempted by the bidders themselves, is useless under very reasonable assumptions on the behavior and the strategies employed by risk-neutral players.

The other effect of the jump bid, which is the inducing and removing of lotteries, causes a similar raise in utility when bidders are risk averse. Two simplified examples were provided to demonstrate the new incentives for the bidders to perform a jump bid. First, knowing that another bidder is risk-averse creates an opportunity to hide information from her and enjoy a less competitive bidding environment (on the average). Second, a risk-averse bidder might want to jump in order to avoid uncertainty in the price (caused by the low-range of the values). We continued by analyzing the different effects of information hiding in a more general environment. Our results suggest that more research might be needed in order to generalize our findings and to better understand the effects of risk aversion in general games with incomplete information.

That being said, we suspect that jump bidding might also be profitable in other more complex environments. The current paper assumed that only one bidder can perform a jump bid, and only as the auction begins. While later jumps can mostly be explained by similar arguments (the only difference lies in the probability distributions of the remaining bidders), jumps by multiple players are definitely a completely different matter. One very specific yet interesting case is the game "Contract Bridge", in which jump bids are very common and come to serve the end of disrupting information flow between the rival players. A more detailed inspection of the game reveals that this is caused by the inapplicability of the monotonicity assumption, namely that the zero-sum nature of the game implies that the utility of a player is negatively correlated to the signal of her rival.[5]

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[5]One could say, in fact, that signals in Contract Bridge are "anti-affiliated", as a strong hand indicates weaker opponents hands. Another observation is that players tend to jump bid more often if their rivals are "vulnerable". This agrees with our analysis, as vulnerable rivals are (at least in some sense) more risk averse.

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A Proofs

A.1 Lemma 3.1

In a symmetric and monotonic equilibrium, a bidder does not win the object unless she receives the highest signal. Focusing on a bidder of type y^T, it follows from continuity of the value function that she should be indifferent between participating or not in the auction. Therefore her average profit can be expressed as:

$$P\left(X_j < \max_k \{X_k\}\right) \cdot 0 + \tag{A.1.1}$$
$$P\left(X_j \geq \max_k \{X_k\}\right) \cdot E\left(v_j - p \,\middle|\, X_j = y^T, X_j \geq \max_k \{X_k\}\right)$$

Since the rightmost condition in (A.1.1) implies that all the players but player j have lower signal than y^T, it follows that the price that player j will pay in that case is exactly T. The uniqueness of y^T is straightforward from (3.1) and the strict monotonicity of the value function. Intuitively, each player regards herself as having the highest signal, and in order for a player to participate in the auction she must be willing to pay at least T.

A.2 Proposition 3.1

Assume (without loss of generality) that the buyers are sorted according to their signals, where player 1 has the highest signal. If $x_1 < y^T$ then in the Cut-T auction the object is not sold, $u_s = T$, which is the minimal utility for the auctioneer in both auction types, which renders the claim trivial. If $x_1 \geq y^T$ and $x_2 < y^T$, then again the object is sold at price T, and again $u_s = T$. Finally, if more than one player participate in the Cut-T auction, then according to the strategies listed above, the revenue for the auctioneer is:

$$\underset{X_{1:N}}{\mathrm{E}}\left[\underset{Y_{1:N}}{\mathrm{E}}\left(v\left(Y_2, Y_{2:N}\right) \middle| \begin{array}{l} Y_1 = X_2, Y_{2:m} = X_{2:m}, \\ y^T > Y_{(m+1):N} \end{array}\right) \middle| \begin{array}{l} X_1 \geq \ldots \geq X_m \geq y^T, \\ y^T > X_{(m+1):N} \end{array}\right]$$

Because the signals are affiliated, this is strictly smaller than:

$$\underset{X_{1:N}}{\mathrm{E}}\left[\underset{Y_{1:N}}{\mathrm{E}}\left(v\left(Y_2, Y_{2:N}\right) \middle| \begin{array}{l} Y_{1:m} = X_{1:m}, \\ y^T > X_{(m+1):N} \end{array}\right) \middle| \begin{array}{l} X_1 \geq \ldots \geq X_m \geq y^T, \\ y^T > X_{(m+1):N} \end{array}\right]$$

And using the law of iterated expectation the last expression equals:

$$\underset{X_{1:N}}{\mathrm{E}}\left[v\left(X_2, X_{2:N}\right) \middle| X_1 \geq \ldots \geq X_m \geq y^T, y^T > X_{(m+1):N}\right] \tag{A.2.1}$$

Expression (A.2.1) is exactly the expected revenue in a standard English auction, given that only m players have signals exceeding y^T. Note that

(A.2.1) is smaller or equals to:

$$\underset{X_{1:N}}{\mathrm{E}}\left[\max\left\{T, v\left(X_2, X_{2:N}\right)\right\}\Big|X_1 \geq \ldots \geq X_m \geq y^T, y^T > X_{(m+1):N}\right] \quad \text{(A.2.2)}$$

Expression (A.2.2) is the revenue in an English auction with a reservation price of T, again, given that only m players have signals exceeding y^T. This is true for every $m \geq 2$ and therefore we get that the revenue in a Cut-T auction is always smaller than the revenue in an English auction with a reservation price of T.[6]

A.3 Proposition 4.1

Splitting the probability space into two classes, we shall see that in each of these classes player 1 has no incentive to jump bid. Assume, without lose of generality, that players $2, \ldots, N$ are sorted according to their signals, i.e. player 2 is the one with the highest signal among players $2, \ldots, N$ and so forth. Denote $s(x_1)$ as the jump bidding function, and since s is strictly monotonic then it is invertible. $s(t)$ is part of player 1's strategy, and therefore $s^{-1}(t)$ can be treated as known to the other players. Also, note that for every jump t there exist a boundary value y^t for the other players' signals, under which the players will not continue to bid after the jump. This value is implicitly defined by:

$$E\left(v_j \middle| X_j = y^t, X_1 = s^{-1}(t), X_j \geq \max_{k \geq 2} X_k\right) - t = 0 \quad \text{(A.3.1)}$$

Case 1: player 2 has a higher signal than player 1. Assume player 1 jumps to the bid of t, and player 2 is the only player to continue bidding after the jump. In that case player 2 continues to bid until the auction reaches her expected value of $E\left(v\left(x_2, s^{-1}(t), X_3, X_4, \ldots, X_N\right)\big|\forall k \geq 3 : X_k \leq y^T\right)$. Because player 1 reveals her value ($s^{-1}(t)$), player 2 makes the correct calculation of her expected value. Thus, if player 1 exceeds that bid then due to the fact that $x_1 < x_2$, and using the single-crossing condition she loses on the average. If more than one player remains after the jump bid, then eventually (because of monotonicity of strategies) only two players will remain, and if player 1 happens to be one of those, then the above analysis holds. However,

[6] This proposition is a slightly extended and specialized form of theorem 21 in Milgrom and Weber (1982, pp. 1116-1117). It can be restated and proved in terms of information.

one might argue that player 1 can profit (on the average) if $x_2 < y^t$, but this does not help. Under this condition, the expected value for player 1 is:

$$EV_1 = E\left(v_1 \left| X_1 = x_1, X_2 < y^T \right.\right) = E\left(v_1 \left| X_1 = s^{-1}(t), X_2 < y^T \right.\right)$$

And from strict monotonicity of value function we get:

$$EV_1 < E\left(v_1 \left| X_1 = s^{-1}(t), X_2 = y^T \right.\right)$$

Using the single crossing condition we now have:

$$EV_1 < E\left(v_1 \left| X_1 = y^T, X_2 = s^{-1}(t) \right.\right) \tag{A.3.2}$$

Finally, using symmetry of value between bidders (ex-ante) and combining (A.3.1) and (A.3.2) we get that $EV_1 < t$, hence jump bidding causes player 1 to lose on average.

Case 2: player 2 has a lower signal than player 1. Given a specific highest signal of x_1, the expected highest bid in the symmetric equilibrium of a regular English auction is (Milgrom and Weber, 1982):

$$EV_2^{EA} = \underset{X_{1:N}}{\mathrm{E}} \left(v\left(X_2, X_{2:N}\right) \left| X_1 = x_1, X_1 > X_2 > \ldots > X_N \right.\right)$$

Player 1's jump bid has two effects: it exposes player 1's high signal, but hides the signal information provided by the players who now decide to stop bidding after the jump bid. Assuming k players continue bidding after the jump bid, the expected highest bid is now:

$$EV_2^{JB} = \underset{X_{1:N}}{\mathrm{E}} \left(\underset{Y_{1:N}}{\mathrm{E}} \left[v\left(Y_2, Y_1, Y_{3:N}\right) \left| \begin{array}{l} Y_{1:k} = X_{1:k}, \\ Y_k > Y_{k+1} > \ldots > Y_N \end{array} \right.\right] \right| \\ X_1 = x_1, X_1 > X_2 > \ldots > X_N \Bigg) \tag{A.3.3}$$

Using the law of iterated expectations:

$$EV_2^{JB} = \underset{X_{1:N}}{\mathrm{E}} \left(\mathrm{E}\left[v\left(X_2, X_1, X_{3:N}\right)\right] \middle| X_1 = x_1, X_1 > X_2 > \ldots > X_N \right)$$

The middle expectation operator is redundant, and so we get:

$$EV_2^{JB} = \underset{X_{1:N}}{\mathrm{E}} \left(v\left(X_2, x_1, X_{3:N}\right) \left| X_1 = x_1, X_1 > X_2 > \ldots > X_N \right.\right)$$

Because of monotonicity of value function we get that the average highest bid in a regular English auction (EV_2^{EA}) is smaller than the average highest bid in an English auction with a jump bid (EV_2^{JB}), and therefore the jump bidding strategy is dominated (ex-ante) by "normal" bidding. Joining together the two cases, it is clear player 1 has no incentive to jump bid under the aforementioned conditions.

A.4 Lemma 4.1

Let us examine a player receiving the signal $\underline{x}_1 = \min\{s^{-1}(t)\}$, we shall see that jumping to t is strictly dominated for this player. Assume, without lose of generality, that player 2 receives the highest signal among players $2, \ldots, N$ and denote by $G(\cdot\,|x_2)$ the probability distribution induced on $s^{-1}(t)$ from $F(\cdot, x_2, \cdot, \ldots, \cdot)$, meaning the probability distribution perceived by player 2 when receiving a signal x_2 and observing a jump of t by player 1. From proposition 1 in Milgrom (1981) it follows that for every $x > y$, $G(\cdot\,|x)$ dominates $G(\cdot\,|y)$ in the sense of first-order stochastic dominance.[7] Therefore, for each jump t there exists (again) a boundary value y^t under which players do not continue bidding after the jump:

$$E\left(v_2 \,\middle|\, X_2 = y^t, X_1 \sim G(\cdot\,|y^t)\right) - t = 0$$

That point being taken care of, and following the lines of proposition 3.1, we now deal with the two cases as before.

Case 1: player 2 has a higher signal than player 1. Similar to the previous proof, if player 2 continues to bid after the jump, then from symmetry and monotonicity (of the non-jumping bidders) she will remain until only two bidders are left, and will reach her expected value, which is strictly higher than:

$$\mathrm{E}\left(v\left(X_2, X_1, X_{3:N}\right) \,\middle|\, X_1 = \underline{x}_1, X_2 > X_1, X_{3:N} \leq y^t\right)$$

(If some bidders continued and then quit the auction, then their values are exactly known to player 2 due to the monotonicity). Now, if $x_2 < y^t$, then in the same manner as described above (using monotonicity):

$$EV_1 = E\left(v_1 \,\middle|\, X_1 = \underline{x}_1, X_2 < y^t\right) < E\left(v_1 \,\middle|\, X_1 = \underline{x}_1, X_2 = y^t\right)$$

[7] Milgrom shows that this holds when the affiliation inequality is strict. We therefore either make our definition of affiliation stronger, or better yet, make use of non-strict first-order stochastic dominance which suffices for our purposes here.

Applying then the single crossing condition:

$$EV_1 < E\left(v_1 \middle| X_1 = y^t, X_2 = \underline{x}_1\right)$$

And finally using symmetry and the definition of $\underline{x}_1$ and y^t we get:

$$EV_1 < E\left(v_2 \middle| X_2 = y^t, X_1 = \underline{x}_1\right) < E\left(v_2 \middle| X_2 = y^t, X_1 \sim G(\cdot|y^t)\right) = t$$

Case 2: player 2 has a lower signal than player 1. The second part of proposition 4.1 holds with only minor changes. Specifically, we change:

$$EV_2^{JB} = \underset{X_{1:N}}{\mathrm{E}}\left(\underset{Y_{1:N}}{\mathrm{E}}\left[v\left(Y_2, Y_1, Y_{3:N}\right)\middle|\begin{array}{l} Y_1 \sim G(\cdot|X_2), Y_{2:k} = X_{2:k}, \\ Y_k > Y_{k+1} > \ldots > Y_N \end{array}\right]\middle| \right.$$
$$\left. X_1 = x_1, \underline{x}_1 > X_2 > \ldots > X_N\right)$$

and then (from definition of $G(\cdot|x_2)$) we change the equality in (A.3.3) to an inequality:

$$EV_2^{JB} \geq \underset{X_{1:N}}{\mathrm{E}}\left(\underset{Y_{1:N}}{\mathrm{E}}\left[v\left(Y_2, Y_1, Y_{3:N}\right)\middle| Y_{1:k} = X_{1:k}, Y_k > Y_{k+1} > \ldots > Y_N\right]\middle| \right.$$
$$\left. X_1 = \underline{x}_1, X_1 > X_2 > \ldots > X_N\right)$$

And the rest of the proof follows.

A.5 Proposition 4.2

Let $s : R \to R$ be a weakly monotonically increasing jumping strategy different than zero, and let $t \neq 0$ be an arbitrary number such that $s^{-1}(t)$ is non-empty. $s^{-1}(t)$ is an interval (could be a point, which is a degenerated interval), and if $s^{-1}(t)$ has a least element then by lemma 4.1 we are done. If not, let $\underline{x}_1 = \inf\left\{s^{-1}(t)\right\}$, and define a new jumping function h:

$$h(x_1) = \begin{cases} t & \text{if } x_1 = \underline{x}_1 \\ s(x_1) & \text{else} \end{cases}$$

h now satisfies the conditions in lemma 4.1, and notice that since F has no singleton atoms the payoffs to the other players do not change, and their previous strategies are therefore best response to h. Applying the same argument presented in lemma 4.1, the expected utility of a player receiving the signal $\underline{x}_1$ and jumping to t is strictly negative. From continuity of the value function, there exists a small enough $\epsilon > 0$ such that $\underline{x}_1 + \epsilon \in s^{-1}(t)$ and the payoff to a player receiving that signal is also strictly negative, which contradicts that her strategy is a best response.

A.6 Observation 5.1

Since u is convex, then by Jensen's inequality it is true that for every $x_{2:m}$:

$$
\begin{aligned}
& u\left[\underset{X_{1:N}}{\mathrm{E}}\left(v\left(X_{1:N}\right)|X_1 = x_2, X_{2:m} = x_{2:m}\right) - \bar{p}\left(x_{2:m}\right)\right] = \\
& u\left[\underset{X_{1:N}}{\mathrm{E}}\left(v\left(X_{1:N}\right) - \bar{p}\left(X_{2:m}\right)|X_1 = x_2, X_{2:m} = x_{2:m}\right)\right] > \\
& \underset{X_{1:N}}{\mathrm{E}}\left(u\left[v\left(X_{1:N}\right) - \bar{p}\left(X_{2:m}\right)\right]|X_1 = x_2, X_{2:m} = x_{2:m}\right) \\
& = 0 = u(0)
\end{aligned}
$$

Where the equality to zero comes from $\bar{p}$ definition. Since u is monotically increasing we get:

$$
\bar{p}(x_{2:m}) < \underset{X_{1:N}}{\mathrm{E}}\left(v\left(X_{1:N}\right)|X_1 = x_2, X_{2:m} = x_{2:m}\right)
$$

It follows immediately that $A(x_1) \geq 0$ for any x_1.

Similarly, using Jensen's inequality again we get for every $x_{1:N}$:

$$
\begin{aligned}
& u\left[v\left(x_{1:n}\right) - \underset{Y_{1:N}}{\mathrm{E}}\left(v\left(Y_{1:N}\right)|Y_1 = x_2, Y_{2:m} = x_{2:m}\right)\right] > \\
& \underset{Y_{1:N}}{\mathrm{E}}\left(u\left[v\left(x_{1:n}\right) - v\left(Y_{1:N}\right)\right]|Y_1 = x_2, Y_{2:m} = x_{2:m}\right)
\end{aligned}
$$

And therefore $B(x_1) \geq 0$ for any x_1.

The non-negativity of C follows from an argument exactly like the one appearing in proposition 3.1, and is therefore omitted.

Finally, note that:

$$
\mathrm{E}\left(U_1^{Non-Jump}|X_1 = x_1\right) = \underset{X_{1:N}}{\mathrm{E}}\left(u\left[v(X_{1:N}) - v(X_{1:N})\right]|X_1 = x_1, X_1 > X_2\right)
$$

$$\mathrm{E}\left(U_1^{Jump}\,|X_1 = x_1\right) = \underset{X_{1:N}}{\mathrm{E}}\left(u\left[v(X_{1:N}) - \bar{p}(X_{2:m})\right]|X_1 = x_1, X_1 > X_2\right)$$

And as $A(x_1)$, $B(x_1)$ and $C(x_1)$ form a (small) telescopic sum, we get

$$\mathrm{E}\left(U_1^{Jump}\,|X_1 = x_1\right) - \mathrm{E}\left(U_1^{Non-Jump}\,|X_1 = x_1\right) \geq 0 \Leftrightarrow A(x_1) + B(x_1) + C(x_1) \geq D(x_1)$$