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OPTIMAL TIES IN CONTESTS

by

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Optimal Ties in Contests

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Abstract

I analyze a mechanism design of a tournament in which the principal can strategically enhance the probability of a tie. The principal decides on a "tie distance" and announces a rule according to which a tie is declared if the difference between the two contestants' performances is within the tie distance. I show that the contestants' equilibrium efforts do not depend on the prizes awarded in case of a tie. I find that there are cases in which the optimal mechanism has a positive tie distance.

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1 Introduction

I analyze one-stage two-player contests with possible tie outcomes. The principal chooses a "tie distance", $d \geq 0$, and does not distinguish between output levels that are within the distance of d from one another. The principal also chooses a prize allocation scheme. I find that in some contests a positive tie distance is optimal. The intuition behind this result is that the principal can "save money" by awarding a smaller sum of prizes in case of a tie.

When Lazear and Rosen (1981) published their groundbreaking paper that broached the topic of contests, they pointed out the following motivation: In labor contracts, workers are often rewarded according to their ranking relative to other workers, rather than simply according to their marginal product. For example, it is likely that the usually high salaries of managers do not reflect their marginal product, but rather serve as a prize incentive for competition among workers in a lower hierarchy. They present a model in which each contestant's observable output is the sum of his effort and a random shock which has a continuous distribution. The implication of this is that the probability of a tie is effectively zero.

However, the mere existence of the concept of ties suggests that the real-world probability of a tie is greater than zero. Students competing for grades often receive identical grades. Sports competitions often end in ties. Even the example cited by Lazear and Rosen (1981) of workers competing for promotions is, in many ways, an example of a competition with a positive probability of a tie: the decision to promote one candidate instead of another is often reported to be arbitrary, since both candidates are "worthy". This is a tie outcome since each of the two applicants is rewarded with the equal prize of a 50% chance to be promoted. Also, many hierarchal firms have midway promotions which are awarded to a group of workers, providing them with the equal prize of a higher salary and the opportunity to compete for further promotions.

Some of the examples above do not strictly demonstrate a positive probability for a tie outcome, but rather demonstrate the vast popularity of "tie" policies. For example, it is possible that the head of the firm has an exact ranking of his workers, but still decides to reward a group of top workers with equal (midway) promotions. Another example is the American grading system, in which a fixed percentage of students receive each grade (A, B, etc.), even if the teacher has a precise ranking of the students' performances. In other words, letter grades are a clear attempt to tie students with similar grades.

While both coarse grading and midway promotions are examples of strategic ties, the reasoning behind their optimality is different. When choosing a grading scheme, the principal is faced with a fixed budget constraint: the utility gained by a student from being ranked above another student is canceled out by the disutility of the other student from being ranked beneath him. The principal's problem is reduced to the maximization of contestants' efforts. Dubey and Geanakoplos (2004) show that, under certain conditions, coarse grading maximizes contestants' efforts. Choosing a wage scheme is different from choosing a grading scheme in that the principal's budget is flexible: The principal is looking to maximize his *profit* which is the sum of efforts minus the expected payment of contestants' salaries. The salaries are chosen by the principal, who has to pay them from his own budget. In this paper I focus on tournaments in which contestants' efforts necessarily decrease with the probability of a tie. I show that a tie may still be part of an optimal mechanism because it allows the principal to reduce the expected sum of wages.

The observation that the organizers of a contest may choose to allocate equal prizes to the few leading contestants relates to the more general problem of prize allocation in contests. Krishna and Morgan (1998) discuss optimal prize allocation in contests of two, three and four contestants, subject to a fixed purse, meaning subject to a constraint that the sum of all prizes is constant. They show that for contests of two or three contestants, the winner-take-all policy is optimal and that in the case of four contestants, an optimal policy allows positive prizes only for the top two rankings, and that the winner's prize is always strictly larger than the runner-up's. Moldevano and Sela (2001) study the case of contests of n contestants that vary in ability, subject to a zero profit constraint. They show that if the cost of effort is concave or linear, a winner-take-all policy is always optimal. Though the prize allocation literature does not relate directly to the topic of ties, it provides strong intuition regarding their inefficiency as a policy: the frequent optimality of winner-take-all contests suggests that a contest with a tie is, at best, a "good approximation" for an optimal contest. In light of this, the result that a tie can be part of an optimal mechanism is particularly surprising.

Independent of this work, Cohen and Sela (2005) analyze a tournament in which there is a positive probability for a tie resulting from discrete efforts and no random shock to output. They focus on one-stage contests as well as multi-stage contests from the mechanism-designer's point of view. Though the topic of that paper is closely related to the topic of this one, the papers are different in an important way: while Cohen and Sela (2005) concentrate on exogenous ties, I analyze a case in which there is not a real tie, but rather a strategic decision to label the outcome a tie when the contestants' output levels are close. There are technical differences as well: Cohen and Sela (2005) assume discrete efforts and no random shock to output, whereas I assume continuous efforts and a continuous shock to output.

A different approach to strategic ties can be found in the discussion of tournaments with midterm reviews (see, for example, Gershkov and Perry (2006)). Though there is no direct discussion of strategic tie breaks or even a model explaining how a tie outcome could come about, there is an interesting and relevant result: in two-stage tournaments with midterm reviews, the contestants will exert a higher effort in the second stage if there is a tie-break in the first stage. Given this result, the principal has an incentive to enhance the probability of a tie in the first stage. Though the focus of the current paper is ties in one-stage tournaments, it provides a setup that could model strategic ties in two stage tournaments by allowing the principal to control the probability of a tie in the first stage through the mechanism of the "tie distance".

The rest of the paper is organized as follows: in section 2, I present the basic setup and assumptions of the model. In section 3, I characterize the optimal mechanism and discuss cases in which the optimal mechanism has a positive tie distance. In section 4, I offer conclusions and discuss possible generalizations.

2 Basic Setup

There are two identical contestants $i = 1, 2$ and a principal. The two contestants are asked to exert non-negative efforts e_1 and e_2 respectively. The principal views a noisy difference in efforts, $z = e_1 - e_2 + x$ where x is a random variable with a symmetric and differentiable density function f and a cumulative density function F, satisfying $xf'(x) \leq 0$ for all x and $E(x) = 0$.

The principal has to decide on a symmetric tournament with a possible tie. The principal's choices variables are:

1. W_1: The prize awarded to the winner
2. W_2: The prize awarded to both contestants in case of a tie
3. W_3: The prize awarded to the loser
4. d: The "tie distance"

If the (noisy) difference between the contestants' output levels is larger than d, a "win" is declared and the winner receives W_1 while the loser receives W_3. If the difference between the contestants' output levels is smaller than d, a tie is declared and both contestants receive W_2. All prizes are constrained to be non-negative.[1] In other words, the principal has to decide on a compensation scheme that is a function $g(z) = (g_1(z), g_2(z))$, where $g_i(z)$ is the payment to contestant i following the outcome z, and $g(z)$ is a of the following form:

[1] It is enough to constrain the prizes to be lower-bounded, and I have decided on 0 as a lower-bound only for simplicity of notation. However, if the prizes are not lower-bounded, the problem is not interesting since the principal can make infinite profits by awarding infinitely negative prizes.

$$g(z) = \begin{cases} (W_1, W_3) & \text{if } z > d; \\ (W_2, W_2) & \text{if } |z| < d; \\ (W_3, W_1) & \text{if } z < -d. \end{cases} \tag{1}$$

The principal is risk neutral and profit maximizing. His profit is given by:

$$\pi(e_1, e_2, W_1, W_2, W_3, d) = e_1 + e_2 - E(payment) \tag{2}$$

The cost of effort, $C(e)$, is strictly convex, twice differentiable and satisfies $C(0) = 0$. The contestants have identical vNM utility functions:

$$U(e_i|e_{\neg i}, W_1, W_2, W_3, d) = \sum_{j=1,2,3} P(j|e_i, e_{\neg i})u(W_j) - C(e_i) \tag{3}$$

Where $P(j|e_i, e_{\neg i})$ denotes the probability that contestant i will win the prize j given his own effort e_i and his opponent's effort $e_{\neg i}$), and u is a monotone function of W_j that satisfies $u(0) = 0$ (no risk preferences are assumed). The contestants choose their effort levels simultaneously and are not aware of the realization of x, though they are familiar with its distribution.

3 Solving for the Optimal Mechanism

I analyze the contestants' behavior in symmetric equilibria, and derive the optimal mechanism that satisfies the restrictions specified in the previous section. Before specifying a sufficient condition for the existence of a symmetric equilibrium, I assume its existence to discuss several important properties.

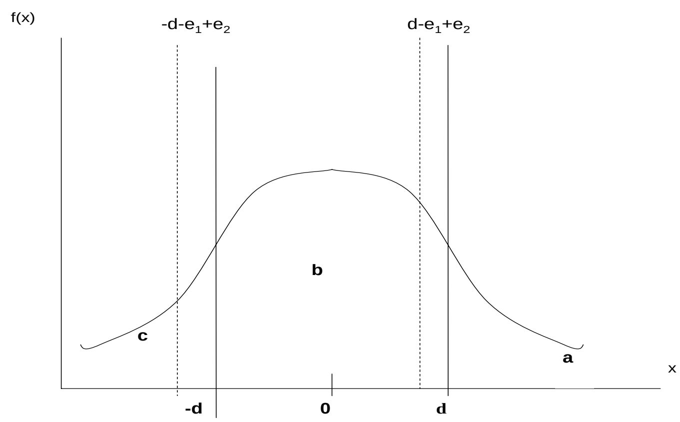
Since the focus is on symmetric equilibria, it is enough to analyze the behavior of contestant 1. Denote:

$$P(z > d|e_1, e_2) = a(e_1, e_2) = a \tag{4}$$

$$P(|z| < d|e_1, e_2) = b(e_1, e_2) = b \tag{5}$$

$$P(z < -d|e_1, e_2) = c(e_1, e_2) = c \tag{6}$$

Graphically, a, b and c are given by:



Where a is the integral from $d - e_1 + e_2$ to ∞, b is the integral from $-d - e_1 + e_2$ to $d - e_1 + e_2$ and c is the integral from $-\infty$ to $-d - e_1 + e_2$ (in this particular illustration, $-e_1 + e_2 < 0$).

Proposition 1 *In a symmetric equilibrium, the contestants' equilibrium efforts do not depend on* W_2.

Proof: See Appendix.

In other words, the contestants will not change their effort levels when promised a higher reward in case of a tie. This result is especially surprising since no risk preferences were assumed. The intuition behind this result is that in a symmetric equilibrium, a marginal increase in effort does nothing to the probability of a tie b (it can easily be seen graphically that the area beneath the curve added on the left when e_1 increases is equal, in limit, to the area subtracted on the right). Therefore, a small deviation will not take into account the award in case of a tie.

Proposition 2 *Any optimal mechanism satisfies* $W_2 = W_3 = 0$.

Proof: See Appendix.

The proof of this proposition is merely looking at the first order conditions of the contestants and the principal. However, the intuition behind the analysis is clear: since the principal is looking to maximize expected profit, a positive reward will be granted only if it results in a higher equilibrium effort level. The effort level is decreasing with W_3, since a higher compensation to the loser decreases incentive to win. According to proposition 1 the equilibrium effort level does not increase with W_2 either. Therefore, the optimal mechanism has to satisfy $W_2 = W_3 = 0$.[2]

Using proposition 2, the contestant's utility function's second order condition can be written compactly. It is written in the appendix and provides a sufficient condition for the existence of a symmetric equilibrium.

The principal is left with the decision of the first prize, W_1, and the tie distance, d. It will be shown that these decisions depend on the specific variables of the problem. The main result of this paper follows:

Proposition 3 *There are contests in which the optimal mechanism satisfies* $W_1 > 0$ *and* $d > 0$*. This is true for contestants of any risk preference.*

Proof: See Appendix.

This proposition is proved by constructing a class of simple examples. However, to understand the intuition, it is easier to refer to the general case. A formal analysis of the general case can be found in the appendix, but the idea is presented here: Increasing d (when W_1 is positive and constant) has two opposing effects on the principal's expected profit. On the one hand, increasing d lowers the expected payment, because it decreases the probability of a win in which the principal has to pay W_1. On the other hand, increasing d decreases the equilibrium effort level,[3] because the expected return for effort decreases with the drop in expected reward.

Though proposition 3 provides economic motivation only for contests with ties that satisfy $W_2 = 0$, continuity implies that there are contests in which a positive probability for a tie and a positive reward in case of a tie guarantee the principal a higher expected profit than the expected profit in case of a no-tie mechanism.

[2]Note that propositions 1 and 2 also hold when d is given exogenously - if the principal is for some reason unable to distinguish between close output levels, he should still reward 0 is case of a tie, and 0 for a loss. This result is consistent with previous literature: when a tie is not allowed, Krishna and Morgan (1998) show that in a two-contestant tournament the optimal compensation in case of a loss is 0. Cohen and Sela (2005) show that when there is an exogenous probability for a tie due to discrete efforts, the optimal reward in case of a tie is 0.

[3]An interesting result follows: if the principal is subject to a fixed purse such as in the Krishna and Morgan (1998) setup, meaning that the sum of all prizes in each scenario must add up to a constant V, then the optimal mechanism is $d = 0$, $W_1 = V$, $W_2 = \frac{V}{2}$ and $W_3 = 0$. This is because the equilibrium effort is decreasing with d for every W_2 (this is immediate from Proposition 1). The principal's problem is equivalent to the maximization of the contestants' equilibrium effort. Notice that the optimality of $d = 0$ is consistent with the findings of Dubey and Geanakoplos (2004): In Dubey and Geanakoplos, the optimality of coarse grading (in the case of homogeneous students) results from stochastic dominance of the output resulting from lower effort over the output resulting from higher effort. The principal can make it less beneficial to choose a low effort by grouping high output levels into a single grade. This argument does not apply in this case because the noise term does not vary with the effort level, so there is no stochastic dominance of low-effort output over high-effort output.

However, the fact that the sum of prizes in case of a tie is lower than the sum of prizes in case of a win is a necessary condition for this outcome.

4 Conclusions and Possible Generalizations

The main result of this paper is that there are tournaments in which the optimal mechanism is not to distinguish between contestants with close output levels. The intuition behind this result is that the principal lowers the expected sum of prizes by increasing the probability of paying nothing in case of a tie. The tradeoff is in the contestants' equilibrium effort level, which decreases with the probability of a tie.

Another result is that the contestants' equilibrium effort level does not depend on the prize awarded in case of a tie. Therefore, the optimal prize in case of a tie must always be 0, or the lowest possible prize.

Though the ideas behind this analysis easily generalize to a contest of n contestants, it is hard to think of a formal generalization since the concept of a tie is hard to generalize. Even when attempting to generalize the problem to three contestants, three different "tie distances" are involved: one for denoting a tie between the first and the second rankings, another for denoting a tie between the second and third rankings and a third to denote a tie among all three. It is easy to see that the problem becomes more complicated as n grows.

A different approach is to generalize the concept of a tie. For instance, one might want to examine the following mechanism: if the two contestants' output levels are within d of one another, reward them with equal prizes. If the distance between their output levels is between d and $2d$, reward them with prizes at the ratios of $1:2$, and so on (if the distance between their output levels is between nd and $(n+1)d$, reward them with prizes of ratio $1:n$). This is a generalization of the concept of a tie since the contestants awards do not depend solely on their rankings, but also on their distance from one another. For a small d, the above policy is close to a policy in which the ratio between the output levels is equal to the ratio between the rewards. It would be interesting to compare this policy to the regular compensation scheme according to piece rates.

A further development of this model is analyzing two-stage tournaments with strategic ties. As mentioned in the Introduction, an important result in the dynamic tournaments literature is that a tie-break in the first stage will increase the contestants' efforts in the second stage. An interesting problem would be to solve for the optimal mechanism while allowing the mechanism designer to decide on a "tie distance".

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A Proof of Proposition 1

The first order condition for maximizing contestant 1's utility is:

$$\frac{\partial a}{\partial e_1}u(W_1) + \frac{\partial b}{\partial e_1}u(W_2) + \frac{\partial c}{\partial e_1}u(W_3) = \frac{\partial C}{\partial e_1} \tag{7}$$

Using the assumption that $e_1 = e_2$, it is easy to see graphically that $\frac{\partial a}{\partial e_1} > 0$, $\frac{\partial b}{\partial e_1} = 0$ and $\frac{\partial c}{\partial e_1} < 0$. Formally, we have:

$$\begin{aligned} \frac{\partial b}{\partial e_1} &= \frac{\partial \int_{-d-e_1+e_2}^{d-e_1+e_2} f(x)dx}{\partial e_1} = \frac{\partial(F(d-e_1+e_2) - F(-d-e_1+e_2))}{\partial e_1} = & (8)\\ &= -f(d-e_1+e_2) + f(d-e_1+e_2) = -f(d) + f(d) = 0 & (9) \end{aligned}$$

Therefore, equation 7 can be rewritten as:

$$\frac{\partial a}{\partial e_1}u(W_1) + \frac{\partial c}{\partial e_1}u(W_3) = \frac{\partial C}{\partial e_1} \tag{10}$$

The solution for e_1 to the above equation does not depend on W_2.

B Proof of Proposition 2

In a similar method to the proof of $\frac{\partial b}{\partial e_1} = 0$ (equation 9), it can be shown that $\frac{\partial c}{\partial e_1} < 0$:

$$\frac{\partial c}{\partial e_1} = \frac{\partial \int_{-\infty}^{-d-e_1+e_2} f(x)dx}{\partial e_1} = \frac{\partial(F(-d-e_1+e_2) - F(-\infty))}{\partial e_1} = \tag{11}$$

$$= -f(-d-e_1+e_2) - 0 = -f(-d) = -f(d) < 0 \tag{12}$$

Recall the principal's profit given by equation 2. An increase in W_1 or in W_2 will obviously increase the expected payment. Seeing that $\frac{\partial b}{\partial e_1} = 0$ and $\frac{\partial c}{\partial e_1} < 0$, by examining the contestant's first order condition (equation 7) and using the assumption that $C(e)$ is strictly convex and $u(\cdot)$ is increasing, it is easy to see that the equilibrium effort level does not increase with W_2 or W_3 (in fact, it does not depend on W_2 and decreases with W_3). Since any increase in W_2 or W_3 will result in an increase in expected payment, and will not result in an increase in the equilibrium effort level, a profit maximizing principal will choose minimal values for W_2 and W_3 - by assumption, these minimal values are 0.

C The Contestant's Second Order Condition

Using proposition 2 and the assumption $u(0) = 0$, the contestant's first order condition (equation 7) can be written as:

$$\frac{\partial a}{\partial e_1} u(W_1) = \frac{\partial C}{\partial e_1} \tag{13}$$

Therefore, the second order condition is:

$$\frac{\partial^2 a}{\partial e_1^2} u(W_1) - \frac{\partial^2 C}{\partial e_1^2} < 0 \tag{14}$$

Using simple analysis similar to that in equations 9 and 12 it can be shown that:

$$\frac{\partial a}{\partial e_1} = f(d - e_1 + e_2) \tag{15}$$

In a symmetric equilibrium:

$$\frac{\partial a}{\partial e_1} = f(d) \tag{16}$$

Therefore, the second order condition can also be written as:

$$- f'(d) - \frac{\partial^2 C}{\partial e_1^2} < 0 \tag{17}$$

D Proof of Proposition 3

I construct a class of examples. The noise term, x, is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$. A contestant's utility from a given prize W is $u(W) = W^\alpha$ ($\alpha > 0$, no specific risk preferences are assumed). The cost of effort is $C(e) = e^2$.

Recalling equation 16:

$$\frac{\partial a}{\partial e_1} = f(d) = 1 \tag{18}$$

Using equation 7 and proposition 2, the first order condition for maximizing the contestant's utility is given by:

$$W_1^\alpha = 2e_1 \Rightarrow e_1 = \frac{W_1^\alpha}{2} \tag{19}$$

It is easy to show that the second order condition (equation 17) holds. However, notice that the above e_1 maximizes $U(e)$ only for e_1 such that $d - e_1 + e_2 \in [-\frac{1}{2}, \frac{1}{2}]$, since otherwise $f(d - e_1 + e_2)$ is 0 and not 1. More accurately, e_1 is given by:

$$e_1 = \begin{cases} \frac{W_1^\alpha}{2} & \text{if } U(e_1 = \frac{W_1^\alpha}{2}) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

Since x is distributed uniformly, the probability of a tie is exactly $2d$ (assuming that the optimal d satisfies $0 \leq d \leq \frac{1}{2}$, which will indeed prove to be the case). Therefore the probability of a win is $1 - 2d$, and the principal's maximization problem is given by:

$$\max_{d, W_1} \quad (\tfrac{W_1^\alpha}{2} - (1 - 2d)W_1) \tag{21}$$

$$s.t. \quad (U(e_1 = \tfrac{W_1^\alpha}{2} | W_1, d) \geq 0) \tag{22}$$

For every W_1 the positive utility constraint is binding, since the principal's expected profit is upwards sloping in d. Therefore the utility constraint determines d:

$$U(e_1 = \frac{W_1^\alpha}{2} | W_1, d) \geq 0 \quad \Leftrightarrow \quad (\frac{1}{2} - d)W_1^\alpha - (\frac{W_1^\alpha}{2})^2 \geq 0 \Leftrightarrow \tag{23}$$

$$\Leftrightarrow \quad (\frac{1}{2} - d) - \frac{W_1^\alpha}{4} \geq 0 \Leftrightarrow \frac{1}{2} - \frac{W_1^\alpha}{4} \geq d \tag{24}$$

It follows that:

$$\frac{1}{2} - \frac{W_1^\alpha}{4} = d \tag{25}$$

The principal's maximization problem (equation 22) is reduced to:

$$\max_{W_1}(\frac{W_1^\alpha}{2} - (1 - 2(\frac{1}{2} - \frac{W_1^\alpha}{4}))W_1) = \max_{W_1}(\frac{1}{2}W_1^\alpha - \frac{1}{2}W_1^{\alpha+1}) \tag{26}$$

The principal's first order condition is given by:

$$\frac{\alpha}{2}W_1^{\alpha-1} - \frac{\alpha+1}{2}W_1^\alpha = 0 \Rightarrow \frac{\alpha}{W_1} - (\alpha+1) = 0 \Rightarrow \frac{\alpha}{\alpha+1} = W_1 \tag{27}$$

Note that this result maximizes the principal's expected profit for every α: W_1 is between 0 and 1 which implies a positive profit, and the expected profit is zero for $W_1 = 0$ and negative for a very large W_1. Moreover, the above analysis shows that there is only one critical point. Therefore the result must be the argument that maximizes the principal's expected profit.

Substituting for W_1 in equation 25 yields $\frac{1}{2} - \frac{(\frac{\alpha}{\alpha+1})^\alpha}{4} = d$. Since $0 < \frac{\alpha}{\alpha+1} < 1$, we obtain that $\frac{1}{4} < d < \frac{1}{2}$.

E Formal Analysis of the Principal's Reduced Maximization Problem

The principal's maximization problem is given by (rewriting equation 2):

$$\max_{d,W_1,W_2,W_3} \quad (2e - 2a(W_1 + W_3) - bW_2) \tag{28}$$

$$s.t. \quad (e = \arg\max(U(e_1|e_2, W_1, W_2, W_3, d)) \tag{29}$$

Using proposition 2 and the assumption $u(0) = 0$, by substituting the constraint with the contestant's utility function (equation 3) the above maximization problem is simplified to:

$$\max_{d,W_1} \quad (2e - 2aW_1) \tag{30}$$

$$s.t. \quad (e = \arg\max(au(W_1) - C(e_1)) \tag{31}$$

Recalling the first order condition (equation 7), the equilibrium effort level satisfies:

$$\frac{\partial a}{\partial e_1}(d)u(W_1) = \frac{\partial C}{\partial e_1} \tag{32}$$

Using equation 16, this can also be written as:

$$f(d)u(W_1) = \frac{\partial C}{\partial e_1} \tag{33}$$

The assumption that $xf'(x) \leq 0$ and $d \geq 0$ implies that $f'(d) \leq 0$ for every d. Therefore, $d = 0$ maximizes the left-hand side. Using the assumption that C is convex, $d = 0$ yields the highest equilibrium effort e. However, the principal's expected profit is also negatively influenced by the probability that he will have to pay the reward W_1. This probability, a, is decreasing with d:

$$\frac{\partial a}{\partial d} = \frac{\partial \int_d^\infty f(x)dx}{\partial d} = \frac{\partial(F(\infty) - F(d))}{\partial d} = 0 - f(d) = -f(d) \leq 0 \tag{34}$$

Therefore, the principal is faced with two conflicting incentives: On the one hand, increasing d will decrease the equilibrium effort. On the other hand, a higher probability for a tie is a lower probability for a win: increasing d will lower the probability of having to pay the first prize, thus lowering the expected payment.