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OPTIMUM COMMODITY TAXATION IN POOLING EQUILIBRIA

by

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Optimum Commodity Taxation in Pooling Equilibria

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Abstract

This paper extends the standard model of optimum commodity taxation (Ramsey (1927) and Diamond-Mirrlees (1971)) to a competitive economy in which some markets are inefficient due to *asymmetric information*. As in most insurance markets, consumers impose varying costs on suppliers but firms cannot associate costs to customers and consequently all are charged equal prices. In a competitive *pooling equilibrium*, the price of each good is equal to average marginal costs weighted by equilibrium quantities. We derive modified *Ramsey-Boiteux Conditions* for optimum taxes in such an economy and show that they include *general-equilibrium effects* which reflect the initial deviations of producer prices from marginal costs, and the response of equilibrium prices to the taxes levied. It is shown that condition on the monotonicity of demand elasticities enables to sign the deviations from the standard formula. The general analysis is applied to the optimum taxation of annuities and life insurance.

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1 Introduction

The setting for the standard theory of optimum commodity taxation (Ramsey (1927), Diamond and Mirrlees (1971), Salanie (2003)) is a competitive equilibrium which attains an efficient resource allocation. In the absence of lump-sum taxes, the government wishes to raise revenue by means of distortive commodity taxes and the theory develops the conditions that have to hold for these taxes to minimize the deadweight loss (the *'Ramsey-Boiteux Conditions'*). The analysis was extended in some directions to allow for an initial inefficient allocation of resources. In such circumstances, aside from the need to raise revenue, taxes/subsidies may serve as means to improve welfare due to market inefficiencies. The rules for optimum commodity taxation, therefore, mix considerations of shifting an inefficient market equilibrium in a welfare enhancing direction and the distortive effects of gaps between consumer and producer marginal valuations generated by commodity taxes.

Two major extensions of the standard model have been explored. First, the inclusion of externalities and the need to finance public goods (Sandmo (1975), Stiglitz and Dasgupta (1971), Lau, Sheshinski and Stiglitz (1978)). While specific assumptions about the form of externalities (e.g. 'atmosphere externalities') or about the form of preferences for public goods (e.g. weak separability), as well as the absence of distributional considerations, were needed to obtain sharper results, these contributions are quite general and the results are robust. The second extension is to allow for imperfect competition (Auerbach and Hines (2001), Guesnerie and Laffont (1978), Myles (1987, 1989)). Here, the results seem to depend more crucially on particular assumptions about the definition of the imperfectly competitive equilibrium (monopoly, Cournot, Bertrand or strategic, repeated game-theoretic, equilibria), about the number of firms in oligopoly markets, about the type of taxes (specific or ad-valoren) and about the presence or absence of uncertainty (making the availability or unavailability of insurance critical). Although these papers provide valuable insights about taxation in specific circumstances, no broad rules on optimum taxation under imperfect competition seem to emerge.

This paper goes in a different direction. Markets are assumed to be perfectly competitive but there is *asymmetric information* between firms and consumers about 'relevant' characteristics which affect the *costs* of firms, as well as consumer preferences. Leading examples are in the field of insurance. Expected costs of medical insurance depend on the health characteristics of the insured. Of course, the value of such insurance to the purchaser depends on the same characteristics. Similarly, the costs of an annuity depend on the expected payout which depends on the individual's survival prospects. Naturally, these prospects also affect the value of an annuity to the individual's expected lifetime utility. Other examples where personal characteristics affect costs are rental contracts (e.g. cars) and fixed-fee contracts for use of certain facilities (clubs).

When firms are able to identify customers' relevant characteristics (in insurance parlance, 'risk class'), competitive pressures equate prices to marginal costs for each customer type, and the competitive equilibrium is efficient. Such identification, however, may not be possible or is imperfect and costly because it requires monitoring of activities, such as quantities purchased (Rothschild-Stiglitz (1976)), and the collection of information available at a multitude of firms. In these circumstances, commodities are sold at the same prices to different types of consumers, mostly to all consumers without distinction. This is called a *pooling-equilibrium.* Zero profits in a competitive pooling equilibrium imply that the price of each good is equal to average marginal costs weighted by the equilibrium quantities purchased by all consumers.

This paper analyses the conditions for optimum commodity taxes in the presence of pooling markets. The modeling of preferences and of costs is general, allowing for any finite number of markets. We focus, though, only on efficiency aspects, disregarding distributional ('equity') considerations[1]. We obtain surprisingly simple modified Ramsey-Boiteux conditions and explain the deviations from the standard model. Broadly, the additional terms that emerge reflect the fact that the initial producer price of each commodity deviates from each consumer's marginal costs, being equal to these costs only on average. Each levied specific tax affects all prices (termed, a *'general-equilibrium effect'*), and, consequently, a small increase in a tax level affects the quantity-weighted gap between producer prices

[1] We have a good idea how exogenous income heterogeneity can be incorporated in the analysis (e.g. Salanie (2003)).

and individual marginal costs, the direction depending on the relation between demand elasticities and costs.

After developing general formulas (Section 3), we analyze (Section 4) an example of a three-good economy with pooling equilibrium in the annuitiy market.

2 Equilibrium With Asymmetric Information

Individuals consume n goods, X_i, $i = 1, 2, .., n$ and a numeraire, Y. There are H individuals whose preferences are characterized by a linearly separable utility function, U

$$U = u^h(\mathbf{x}^h,\ \alpha) + y^h, \quad h = 1, 2, .., H \tag{1}$$

where $\mathbf{x}^h = (x_1^h, x_2^h, .., x_n^h,)$, x_i^h is the quantity of good i and y^h is the quantity of the numeraire consumed by individual h. The utility function, u^h, is assumed to be strictly concave and differentiable in $\mathbf{x}^h$. Linear separability is assumed to eliminate distributional considerations, focusing on the efficiency aspects of optimum taxation. It is well-known how to incorporate equity issues in the analysis of commodity taxation (e.g. Salanie (2003)).

The parameter α is a personal attribute which is singled out because it has *cost effects*. Specifically, it is assumed that the unit costs of good i consumed by individuals with a given α ('type α') is $c_i(\alpha)$. Leading examples are health and longevity insurance. The health status of an individual affects both his consumption preferences and the costs to the medical insurance provider. Similarly, the payout of annuities (e.g. retirement benefits) is contingent on survival and hence depends on the individual's relevant mortality function. Other examples are car rentals and car insurance, whose costs and value to consumers depend on driving patterns and other personal characteristics[2].

It is assumed that α is continuously distributed in the population, with a distribution function, $F(\alpha)$, over a finite interval, $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$.

The economy has given total resources, $R > 0$. With unit costs of 1 for the

[2]Representation of these characteristics by a single parameter is, of course, a simplification.

numeraire, Y, the aggregate resource constraint is written

$$\int_{\underline{\alpha}}^{\bar{\alpha}} [\mathbf{c}(\alpha)\mathbf{x}(\alpha) + y(\alpha)]dF(\alpha) = R \tag{2}$$

where $\mathbf{c}(\alpha) = (c_1(\alpha), c_2(\alpha), .., c_n(\alpha))$, $\mathbf{x}(\alpha) = (x_1(\alpha), x_2(\alpha), .., x_n(\alpha))$, $x_i(\alpha)$ being the aggregate quantity of X_i consumed by all type α individuals: $x_i(\alpha) = \sum_{h=1}^{H} x_i^h(\alpha)$ and, correspondingly, $y(\alpha) = \sum_{h=1}^{H} y^h(\alpha)$.

The First-Best allocation is obtained by maximization of a utilitarian welfare function, W,

$$W = \int_{\underline{\alpha}}^{\bar{\alpha}} \left[\sum_{h=1}^{H} (u^h(\mathbf{x}^h; \alpha) + y^h) \right] dF(\alpha) \tag{3}$$

s.t. the resource constraint (2). The F.O.C. for an interior solution equates marginal utilities and costs for all individuals of the same type. That is, for each α,

$$u_i^h(\mathbf{x}^h; \alpha) - c_i(\alpha) = 0, \quad i = 1, 2, .., n \quad h = 1, 2, .., H \tag{4}$$

where $u_i^h = \dfrac{\partial u^h}{\partial x_i}$. The unique solution to (4), denoted $\mathbf{x}^{*h}(\alpha) = (x_1^{*h}(\alpha), x_2^{*h}(\alpha)..,$ $x_n^{*h}(\alpha))$, and the corresponding total consumption of type α individuals $\mathbf{x}^h(\alpha) =$ $(x_1^*(\alpha), x_2^*(\alpha)..,\ x_n^*(\alpha))$, $x_i^h(\alpha) = \sum_{h=1}^{H} x_i^h(\alpha)$. Individuals' optimum level of the numeraire Y (and hence utility levels) is indeterminate, but the total amount, y^*, is determined by the resource constraint, $y^* = R - \int_{\underline{\alpha}}^{\bar{\alpha}} \mathbf{c}(\alpha)\mathbf{x}^*(\alpha)dF(\alpha)$.

The *First-Best* allocation can be supported by competitive markets with individualized prices equal to marginal costs[3]. That is, if p_i is the price of good i, then efficiency is attained when all type α individuals face the same price $p_i(\alpha) = c_i(\alpha)$.

When α is *private information* unknown to suppliers (and not verifiable by monitoring individuals' purchases), then for each good firms will charge the same price to all individuals. This is called a (Second-Best) *Pooling Equilibrium.*

[3]The only constraint on the allocation of incomes, $m^h(\alpha)$, is that they support an interior solution. The modifications required to allow for zero equilibrium quantities are well-known and immaterial for the following.

Pooling Equilibrium

Good X_i is offered at a price p_i to all individuals, $i = 1, 2, .., n$. The competitive price of the numeraire is 1. Individuals maximize their utility, (1), subject to the budget constraint

$$\mathbf{p}\mathbf{x}^h + y^h = m^h, \quad h = 1, 2, .., H \tag{5}$$

where $m^h = m^h(\alpha)$ is the (given) income of the h-th type α individual. It assumed that for all α, the level of m^h yields interior solutions. The F.O.C. are

$$u_i^h(\mathbf{x}^h; \alpha) - p_i = 0, \quad i = 1, 2, .., n \quad h = 1, 2, .., H \tag{6}$$

the unique solutions to (6) are the *compensated demand functions* $\hat{\mathbf{x}}^h(\mathbf{p}; \alpha) = \left(\hat{x}_1^h(\mathbf{p}; \alpha), \hat{x}_2^h(\mathbf{p}; \alpha), .., \hat{x}_n^h(\mathbf{p}; \alpha)\right)$, and the corresponding type α total demands $\hat{\mathbf{x}}(\mathbf{p}; \alpha) = \sum\limits_{h=1}^{H} \hat{\mathbf{x}}^h(\mathbf{p};\alpha)$. The optimum levels of Y, $\hat{y}^h$, are obtained from the budget constraints (5): $\hat{y}^h(\mathbf{p}; \alpha) = m^h(\alpha) - \mathbf{p}\hat{\mathbf{x}}^h(\mathbf{p}; \alpha)$, with total consumption of $\hat{y}(\mathbf{p}; \alpha) = \sum\limits_{h=1}^{H} \hat{y}^h = \sum\limits_{h=1}^{H} m^h(\alpha) - \mathbf{p}\hat{\mathbf{x}}(\mathbf{p}; \alpha)$. The economy is closed by the identity $R = \sum\limits_{h=1}^{H} m^h(\alpha)$.

Let $\pi_i(\mathbf{p})$ be total profits in the production of good i:

$$\pi_i(\mathbf{p}) = p_i\hat{x}_i(\mathbf{p}) - \int_{\underline{\alpha}}^{\bar{\alpha}} c_i(\alpha)\hat{x}_i(\mathbf{p}; \alpha) dF(\alpha) \tag{7}$$

where $\hat{x}_i(\mathbf{p}) = \int_{\underline{\alpha}}^{\bar{\alpha}} \hat{x}_i(\mathbf{p}; \alpha) dF(\alpha)$ is the *aggregate demand* for good i.

Definition 1 [4] *A pooling-equilibrium is a vector of prices, $\hat{\mathbf{p}}$, which satisfies $\pi_i(\hat{\mathbf{p}}) = 0$, $i = 1, 2, .., n$ or*

$$\hat{p}_i = \frac{\int_{\underline{\alpha}}^{\bar{\alpha}} c_i(\alpha)\hat{x}_i(\hat{\mathbf{p}}; \alpha) dF(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} \hat{x}_i(\hat{\mathbf{p}}; \alpha) dF(\alpha)}, \quad i = 1, 2, .., n. \tag{8}$$

[4] For general analyses of pooling equilibria see, for example, Laffont and Martimort (2002) and Salanie (1997).

Equilibrium prices are weighted averages of marginal costs, the weights being the equilibrium quantities purchased by the different α types. Writing (7) (or (8)) in matrix form:

$$\boldsymbol{\pi}(\hat{\mathbf{p}}) = \hat{\mathbf{p}}X(\hat{\mathbf{p}}) - \int_{\underline{\alpha}}^{\bar{\alpha}} \mathbf{c}(\alpha)\hat{X}(\hat{\mathbf{p}};\alpha)dF(\alpha) = \mathbf{0} \tag{9}$$

where $\boldsymbol{\pi}(\hat{\mathbf{p}}) = (\pi_1(\hat{\mathbf{p}}), \pi_2(\hat{\mathbf{p}}), .., \pi_n(\hat{\mathbf{p}}))$,

$$\hat{\mathrm{X}}(\hat{\mathbf{p}};\alpha) = \begin{bmatrix} \hat{x}_1(\hat{\mathbf{p}};\alpha) & & 0 \\ & \ddots & \\ 0 & & \hat{x}_n(\hat{\mathbf{p}};\alpha) \end{bmatrix}, \tag{10}$$

$\hat{X}(\hat{\mathbf{p}}) = \int_{\underline{\alpha}}^{\bar{\alpha}} X(\hat{\mathbf{p}};\alpha)dF(\alpha)$, $\mathbf{c}(\alpha) = (c_1(\alpha), c_2(\alpha), .., c_n(\alpha))$, and $\mathbf{0}$ is $1 \times n$ zero vector $\mathbf{0} = (0, 0, .., 0)$. Let $\hat{K}(\hat{\mathbf{p}})$ be the $n \times n$ matrix with elements $\hat{k}_{ij}$,

$$\hat{k}_{ij}(\hat{\mathbf{p}}) = \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_i - c_i(\alpha))s_{ij}(\hat{\mathbf{p}};\alpha)dF(\alpha), \quad i.j = 1, 2, .., n \tag{11}$$

where $s_{ij}(\hat{\mathbf{p}};\alpha) = \dfrac{\partial \hat{x}_i(\hat{\mathbf{p}};\alpha)}{\partial p_j}$ are the substitution terms.

We can now state:

Proposition 1. *When $\hat{X}(\mathbf{p}) + \hat{K}(\mathbf{p})$ is positive-definite for any $\mathbf{p}$, then there exist unique and globally stable prices, $\hat{\mathbf{p}}$, which satisfy (9).*

Proof. Appendix A.

We shall assume throughout that the condition in Proposition 1 is satisfied. Note that when costs are independent of α, $\hat{p}_i - c_i = 0$, $i = 1, 2, .., n$, $\hat{K} = 0$ and this condition is trivially satisfied.

3 Optimum Commodity Taxation

Suppose that the government wishes to impose specific commodity taxes on $\mathbf{X}_i$, $i = 1, 2, .., n$. Let the unit tax (subsidy) on $\mathbf{X}_i$ be t_i so that its (tax inclusive) consumer price is $q_i = p_i + t_i$, $i = 1, 2, .., n$. Consumer demands, $\hat{x}_i^h(\mathbf{q}; \alpha)$, are now functions of these prices, $\mathbf{q} = \mathbf{p} + \mathbf{t}$, $\mathbf{t} = (t_1, t_2, .., t_n)$. Correspondingly, total demands for each good by type α individuals is $\hat{x}_i(\mathbf{q}; \alpha) = \sum_{h=1}^{H} \hat{x}_i^h(\mathbf{q}; \alpha)$.

As before, the equilibrium vector of consumer prices, $\hat{\mathbf{q}}$, is determined by zero-profits conditions:

$$\hat{q}_i = \frac{\int_{\underline{\alpha}}^{\bar{\alpha}} (c_i(\alpha) + t_i)\hat{x}_i(\hat{\mathbf{q}}; \alpha) dF(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} \hat{x}_i(\hat{\mathbf{q}}; \alpha) dF(\alpha)} \qquad i = 1, 2, .., n \tag{12}$$

or, in matrix form,

$$\boldsymbol{\pi}(\hat{\mathbf{q}}) = \hat{\mathbf{q}}\hat{\mathbf{X}}(\hat{\mathbf{q}}) - \int_{\underline{\alpha}}^{\bar{\alpha}} (\mathbf{c}(\alpha) + \mathbf{t})\hat{\mathbf{X}}(\hat{\mathbf{q}}; \alpha) dF(\alpha) = \mathbf{0} \tag{13}$$

where $\hat{\mathbf{X}}(\hat{\mathbf{q}}; \alpha)$ and $\mathbf{X}(\hat{\mathbf{q}})$ are the diagonal $n \times n$ matrices defined above, with $\hat{\mathbf{q}}$ replacing $\hat{\mathbf{p}}$.

Note that each element in $\hat{K}(\hat{\mathbf{q}})$, $k_{ij}(\hat{\mathbf{q}}) = \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_i - c_i(\alpha)) s_{ij}(\hat{\mathbf{q}}; \alpha) dF(\alpha)$, also depends on $\hat{p}_i$ or $\hat{q}_i - t_i$. It is assumed that $\hat{X}(\mathbf{q}) + \hat{K}(\mathbf{q})$ is positive definite for all $\mathbf{q}$. Hence, given $\mathbf{t}$, there exist unique prices, $\hat{\mathbf{q}}$ (and the corresponding $\hat{\mathbf{p}} = \hat{\mathbf{q}} - \mathbf{t}$), which satisfy (13).

Observe that each equilibrium price, $\hat{q}_i$, depends on the whole vector of tax rates, $\mathbf{t}$. Specifically, differentiating (13) w.r.t. the tax rates, we obtain:

$$(\hat{X}(\hat{\mathbf{q}}) + \hat{K}(\hat{\mathbf{q}}))\hat{Q} = \hat{X}(\hat{\mathbf{q}}) \tag{14}$$

where $\hat{Q}$ is the $n \times n$ matrix whose elements are $\dfrac{\partial \hat{q}_i}{\partial t_j}$, $i, j = 1, 2, .., n$.

All principal minors of $\hat{X} + \hat{K}$ are positive and it has a well-defined inverse. Hence, from (14),

$$\hat{Q} = (\hat{X} + \hat{K})^{-1}\hat{X}. \tag{15}$$

It is seen from (15) that equilibrium consumer prices rise w.r.t. an increase in own tax rates:

$$\frac{\partial \hat{q}_i}{\partial t_i} = \hat{x}_i(\hat{\mathbf{q}}) \frac{\left|\hat{X} + \hat{K}\right|_{ii}}{\left|\hat{X} + \hat{K}\right|} \tag{16}$$

where $\left|\hat{X}+\hat{K}\right|$ is the determinant of $\hat{X}+\hat{K}$, and $\left|\hat{X}+\hat{K}\right|_{ii}$ is the principal minor obtained by deleting the i-th row and the i-th column. In general, the sign of cross-price effects due to tax rate increases is indeterminate, depending on substitution and complementarity terms.

We also deduce from (15) that, as expected, $\hat{K}=0$, $\frac{\partial \hat{q}_i}{\partial t_i}=1$ and $\frac{\partial \hat{q}_i}{\partial t_j}=0$, $i \neq j$, when costs in all markets are independent of customer type (no asymmetric information), that is, the initial equilibrium is efficient: $p_i - c_i = 0$, $i = 1, 2, .., n$.

From (1) and (3), social welfare in the pooling equilibrium is written

$$W(\mathbf{t}) = \int_{\underline{\alpha}}^{\bar{\alpha}} \left[\sum_{h=1}^{H} u^h(\hat{\mathbf{x}}^h(\hat{\mathbf{q}};\alpha)) - \mathbf{c}(\alpha)\hat{\mathbf{x}}(\hat{\mathbf{q}};\alpha) \right] dF(\alpha) + R \tag{17}$$

The problem of optimum commodity taxation can now be stated: the government wishes to raise a given amount, T, of tax revenue:

$$\mathbf{t}\hat{\mathbf{x}}(\hat{\mathbf{q}}) = T \tag{18}$$

by means of unit taxes $\mathbf{t} = (t_1, t_2, .., t_n)$ that maximize $W(\mathbf{t})$.

Maximization of (17) s.t. (18) and (15) yields, after substitution of $u_i^h - q_i = 0$, $i = 1, 2, .., n$, $h = 1, 2, .., H$ from the individual F.O.C., that optimum tax levels, denoted $\hat{\mathbf{t}}$, satisfy:

$$(1+\lambda)\hat{\mathbf{t}}\hat{S}\hat{Q} + \mathbf{1}\hat{K}\hat{Q} = -\lambda \mathbf{1}\hat{X} \tag{19}$$

where $\hat{S}$ is the $n \times n$ aggregate substitution matrix whose elements are $s_{ij}(\hat{\mathbf{q}}) = \int_{\underline{\alpha}}^{\bar{\alpha}} s_{ij}(\hat{\mathbf{q}};\alpha)dF(\alpha)$, $\mathbf{1}$ is the $1 \times n$ unit vector, $\mathbf{1} = (1, 1, .., 1)$, and $\lambda > 0$ is the *Lagrange multiplier* of (18).

Rewrite (19) in the more familiar form:

$$\hat{\mathbf{t}}S = -\frac{1}{1+\lambda}\left[\mathbf{1}(\lambda\hat{X} + \hat{K}\hat{Q})\hat{Q}^{-1}\right]$$

substituting from (15)

$$= \frac{\lambda}{1+\lambda}\mathbf{1}\hat{X} - \mathbf{1}\hat{K} \tag{20}$$

Equation (20) is our fundamental result. Let's examine these optimality conditions w.r.t. a particular tax, t_i :

$$\sum_{j=1}^{n} \hat{t}_j s_{ji}(\hat{\mathbf{q}}) = -\frac{\lambda}{1+\lambda}\hat{x}_i(\hat{\mathbf{q}}) - \sum_{j=1}^{n} \hat{k}_{ji} \tag{21}$$

Denoting aggregate demand elasticities by $\varepsilon_{ij} = \varepsilon_{ij}(\mathbf{q}) = \dfrac{\mathbf{q}_j s_{ij}(\mathbf{q})}{\hat{x}_i(\mathbf{q})}$, $i, j = 1, 2, .., n$, and using symmetry, $s_{ij}(\mathbf{q}) = s_{ji}(\hat{\mathbf{q}})$, (21) can be rewritten in elasticity form:

$$\sum_{j=1}^{n} \hat{t}'_j s_{ji}(\hat{\mathbf{q}}) = -\theta - \sum_{j=1}^{n} \hat{k}'_{ji} \tag{22}$$

where $\hat{t}'_j = \hat{t}_j / \hat{q}_j$, $j = 1, 2, .., n$ are the optimum ratios of taxes to consumer prices, $\theta = \dfrac{\lambda}{1+\lambda}$,

$$\hat{k}'_{ji} = \frac{1}{\hat{\mathbf{q}}_i} \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_j - c_j) \hat{x}_j(\hat{\mathbf{q}}; \alpha) \varepsilon_{ji}(\hat{\mathbf{q}}; \alpha) dF(\alpha) \tag{23}$$

where $\varepsilon_{ji}(\hat{\mathbf{q}}; \alpha) = \dfrac{\hat{q}_i s_{ji}(\hat{\mathbf{q}}; \alpha)}{x_j(\hat{\mathbf{q}}; \alpha)}$, $i, j = 1, 2, .., n$.

Compared to the standard case, $\hat{k}_{ji} = \hat{k}'_{ji} = 0$, $i, j = 1, 2, .., n$, the modified Ramsey-Boiteux Conditions (21) or (22), have the additional term, $\sum_{j=1}^{n} \hat{k}_{ji}$ or $\sum_{j=1}^{n} \hat{k}'_{ji}$, respectively, on the R.H.S. The interpretation of this term is straightforward.

In a pooling equilibrium, prices are a weighted average of marginal costs, the weights being the equilibrium quantities, (9). Since demands, in general, depend on all prices, all equilibrium prices are interdependent. It follows that an increase in the unit tax of any good affects *all* equilibrium (producer and consumer) prices. This *general-equilibrium effect* of a specific tax is present also in perfectly competitive economies with non-linear technologies, but these price effects have no first-order welfare effects because of the equality of prices and marginal costs. In contrast, in a pooling equilibrium, where prices deviate from marginal costs (being equal to the latter only on average), there is a first-order welfare implication. The term $\hat{k}_{ji} = \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_j - c_j(\alpha)) s_{ij}(\hat{\mathbf{q}}; \alpha) dF(\alpha)$ (or the equivalent term $\hat{k}'_{ji}$) is a welfare loss (< 0) or gain (> 0) equal to the difference between the producer price and the marginal costs of type α individuals, positive or negative, times the change in the quantity of good j due to an increase in the price of good i. As we shall show below, the sign of $\hat{k}_{ji}$ (or $\hat{k}'_{ji}$) depends on the relation between demand elasticity and α.

As seen from (21) or (22), the signs of $\sum_{j=1}^{n} \hat{k}_{ji}$, respectively, $i = 1, 2, .., n$ determine the direction that optimum taxes in a pooling equilibrium differ from those taxes in an initially efficient equilibrium. We shall now show that the sign of these terms depends on the relation between demand elasticities and costs.

Proposition 2. $\hat{k}'_{ji} > 0$ (< 0) *when* ε_{ji} *increases (decreases) with* α.

Proof. Appendix B.

An implication of Proposition 2 is that when all elasticities ε_{ji} are constant, then $\hat{k}'_{ji} = 0$, $i, j = 1, 2, .., n$, (20) or (21) become the standard Ramsey-Boiteux Conditions, solving for the *same* optimum tax structure.

The intuition for the condition in Proposition 1 is the following: $\hat{k}_{ji} < 0$ means that profits of good j fall as q_i increases, calling for an increase in the equilibrium price of good j. This 'negative' effect due to the pooling equilibrium leads, by (20), to a smaller tax on good i compared to the standard case. Of course, this conclusion holds only if this effect has the same sign when summing over all markets, $\sum_{j=1}^{n} k_{ji} < 0$. The opposite conclusion follows when $\sum_{j=1}^{n} k_{ji} > 0$.

4 Example: Taxation of Annuities

Consider individuals who consume three goods: annuities, life insurance and a numeraire. Each annuity pays \$1 to the holder as long as he lives. Each unit of life insurance pays \$1 upon death of the policy owner. There is one representative individual and for simplicity let expected utility, U, be separable and have no time preference:

$$U = u(a)z + v(b) + y \tag{24}$$

where a is the amount of annuities, z is expected lifetime, b the amount of life insurance and y the amount of numeraire. Utility of consumption, u, and the utility from bequests, v, are assumed to be strictly concave. As previously, we assume that the equilibrium values of all variables are strictly positive.

Individuals are differentiated by their survival prospects. Let α represent an individual's 'risk-class' ('type α') $z = z(\alpha)$, z strictly increasing in α. α is taken to be continuously distributed in the population over the interval $\underline{\alpha} \le \alpha \le \bar{\alpha}$, with a distribution function, $F(\alpha)$. Accordingly, average lifetime in the population is $\bar{z} = \int_{\underline{\alpha}}^{\bar{\alpha}} z(\alpha)dF(\alpha)$.

Assume a zero rate of interest. In a full information competitive equilibrium, the price of an annuity to type α individuals is $z(\alpha)$ and the prices of life insurance and of the numeraire are 1. All individuals will purchase the same amount of annuities and life insurance and, for a given income, optimum utility increases with life expectancy, $z(\alpha)$.

Let p_a and p_b be the prices of annuities and life insurance, respectively, in a pooling equilibrium. Individuals' budget constraints are:

$$p_a a + p_b b + y = m \tag{25}$$

Maximization of (24) s.t. (25), yields (compensated) demand functions $\hat{a}(p_a, p_b; \alpha)$ and $\hat{b}(p_a, p_b; \alpha)$, while $\hat{y} = m - p_a\hat{a} - p_b\hat{b}$. Profits of the two goods, π_a and π_b, are:

$$\begin{aligned}
\pi_a(p_a,\ p_b) &= \int_{\underline{\alpha}}^{\bar{\alpha}} (p_a - z(\alpha))\hat{a}(p_a, p_b; \alpha)dF(\alpha) \\
\pi_b(p_a,\ p_b) &= \int_{\underline{\alpha}}^{\bar{\alpha}} (p_b - 1)\hat{b}(p_a, p_b; \alpha)dF(\alpha)
\end{aligned} \tag{26}$$

Equilibrium prices, denoted $\hat{p}_a$ and $\hat{p}_b$, are implicitly determined by $\pi_a = \pi_b = 0$. Clearly, $\hat{p}_b = 1$ (since 1 is the unit cost for all individuals).

Aggregate quantities of annuities and life insurance are $\hat{a}(p_a, p_b) = \int_{\underline{\alpha}}^{\bar{\alpha}} \hat{a}(p_a, p_b; \alpha)dF(\alpha)$ and $\hat{b}(p_a, p_b) = \int_{\underline{\alpha}}^{\bar{\alpha}} \hat{b}(p_a, p_b; \alpha)dF(\alpha)$, respectively. We assume (Appendix A) that:

$$\begin{aligned}
&\hat{a}(p_a, p_b) + \hat{k}_{11} > 0, \quad \hat{b}(p_a, p_b) + \hat{k}_{22} > 0 \\
\text{and}& \\
&\left(\hat{a}(p_a, p_b) + \hat{k}_{11}\right)\left(\hat{b}(p_a, p_b) + \hat{k}_{22}\right) - \hat{k}_{12}\hat{k}_{21} > 0
\end{aligned} \tag{27}$$

where[5]

$$\hat{k}_{1i} = \int_{\underline{\alpha}}^{\bar{\alpha}} (p_a - z(\alpha)) s_{1i} dF(\alpha), \quad s_{1i} = \frac{\partial \hat{a}(p_a, p_b; \alpha)}{\partial p_i}, \quad i = a, b$$

and (28)

$$\hat{k}_{2i} = \int_{\underline{\alpha}}^{\bar{\alpha}} (p_b - 1) s_{2i} dF(\alpha), \quad s_{2i} = \frac{\partial \hat{b}(p_a, p_b; \alpha)}{\partial p_i}, \quad i = a, b$$

As seen in Figure 1 (drawn for the case $k_{12} > 0$), the pooling equilibrium $(\hat{p}_a, \hat{p}_b = 1)$ is unique and stable.

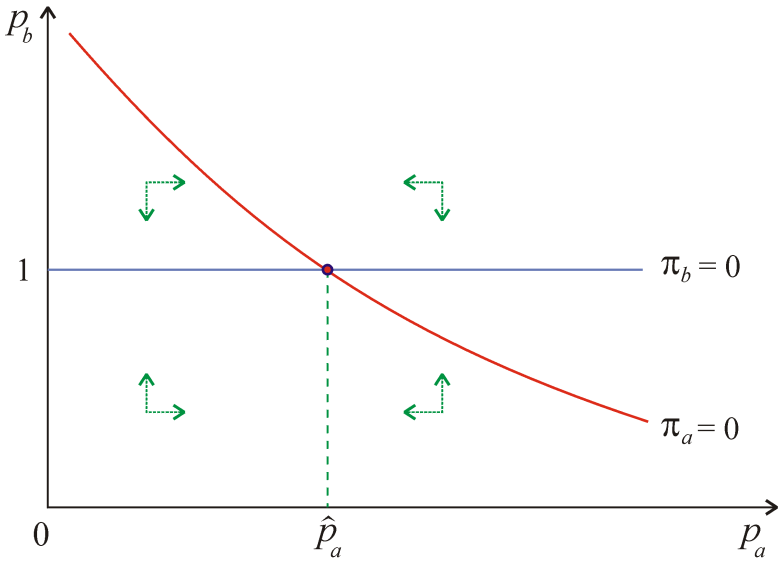

Figure 1

Now consider unit taxes, t_a and t_b, imposed on annuities and life insurance with consumer prices denoted $q_a = p_a + t_a$ and $q_b = p_b + t_b$, respectively. Applying the optimality conditions (21), optimum taxes, $(\hat{t}_a,\ \hat{t}_b)$, satisfy the conditions:

$$s_{11}\hat{t}_a + s_{21}\hat{t}_b = -\theta\hat{a}(\hat{q}_a, \hat{q}_b) - k_{11}$$

(29)

$$s_{12}\hat{t}_a + s_{22}\hat{t}_b = -\theta$$

where $0 < \theta < 1$, $s_{ij}(\hat{q}_a, \hat{q}_b) = \int_{\underline{\alpha}}^{\bar{\alpha}} s_{ij}(\hat{q}_a, \hat{q}_b; \alpha) dF(\alpha)$, $s_{1i}(\hat{q}_a, \hat{q}_b; \alpha) = \dfrac{\partial \hat{a}(\hat{q}_a, \hat{q}_b; \alpha)}{\partial q_i}$, $s_{2i}(\hat{q}_a, \hat{q}_b; \alpha) = \dfrac{\partial \hat{b}(\hat{q}_a, \hat{q}_b; \alpha)}{\partial q_i}$, $i = a, b$, and $\hat{k}_{11} = \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_a - z(\alpha)) s_{11}(\hat{q}_a, \hat{q}_b; \alpha) dF(\alpha)$.

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[5]By concavity and separability, (24), $s_{11} < 0$, $s_{22} < 0$ and $s_{12}, s_{21} > 0$.

Equations (29) are the modified Ramsey-Boiteux Conditions for the case of one pooling-market.

To see in what direction the pooling equilibrium affects optimum taxes, write (29) in elasticity form, using symmetry $s_{ij} = s_{ji}$, $\varepsilon_{11} = \dfrac{\hat{q}_a s_{11}}{\hat{a}}$, $\varepsilon_{12} = \dfrac{\hat{q}_a s_{12}}{\hat{a}}$, $\varepsilon_{21} = \dfrac{\hat{q}_b s_{21}}{\hat{b}}$, $\varepsilon_{22} = \dfrac{\hat{q}_b s_{22}}{\hat{b}}$:

$$\varepsilon_{11}\hat{t}'_a + \varepsilon_{12}\hat{t}'_b = -\theta - \frac{\hat{k}_{11}}{\hat{a}}$$

$$\varepsilon_{21}\hat{t}'_a + \varepsilon_{22}\hat{t}'_b = -\theta \tag{30}$$

where $\hat{t}'_a = \dfrac{\hat{t}_a}{\hat{q}_a}$ and $\hat{t}'_b = \dfrac{t_b}{q_b}$ are the ratios of optimum taxes to consumer prices. Solving (30) for the tax rates, using the identities $\varepsilon_{i0} + \varepsilon_{i1} + \varepsilon_{i2} = 0$, $i = 1, 2$, where 0 denotes the untaxed numeraire:

$$\frac{\hat{t}'_a}{\hat{t}'_b} = \frac{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{10} + \dfrac{\hat{k}_{11}}{\theta\hat{a}}}{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{20} - \dfrac{\hat{k}_{11}}{\theta\hat{a}}} \tag{31}$$

We know that optimum tax ratios depend on complementarity or substitution of the taxed goods with the untaxed good, ε_{i0}, $i = 1, 2$. The additional term, due to the pooling equilibrium in the annuity market, is $\dfrac{\hat{k}_{11}}{\theta\hat{a}}$, which may be negative or positive. Proposition 2 states that $\hat{k}_{11} < 0$ when the elasticity of the demand for annuities increases with life expectancy, $z(\alpha)$. Observe that a higher $z(\alpha)$ increases the amount of annuities purchased, $\dfrac{\partial\hat{a}}{\partial\alpha} > 0$. Hence, under this assumption, the additional term tends to (relatively) reduce the tax on annuities. The opposite argument applies when $\hat{k}_{11} > 0$.

Appendix A

An interior pooling equilibrium, $\hat{\mathbf{p}}$, is defined by the system of equations

$$\boldsymbol{\pi}(\hat{\mathbf{p}}) = \hat{\mathbf{p}}\hat{\mathbf{X}}(\hat{\mathbf{p}}) - \int_{\underline{\alpha}}^{\bar{\alpha}} \mathbf{c}(\alpha)\hat{\mathbf{X}}(\hat{\mathbf{p}};\alpha)dF(\alpha) = \mathbf{0} \tag{A.1}$$

where $\boldsymbol{\pi}(\hat{\mathbf{p}}) = (\pi_1(\hat{\mathbf{p}}), \pi_2(\hat{\mathbf{p}}), .., \pi_n(\hat{\mathbf{p}}))$, $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2, .., \hat{p}_n)$, $\hat{X}(\hat{\mathbf{p}})$ is the diagonal $n \times n$ matrix:

$$\hat{\mathrm{X}}(\hat{\mathbf{p}}) = \begin{bmatrix} \hat{x}_1(\hat{\mathbf{p}}) & & 0 \\ & \ddots & \\ 0 & & \hat{x}_n(\hat{\mathbf{p}}) \end{bmatrix}, \tag{A.2}$$

while $X(\mathbf{p};\alpha)$ is the diagonal $n \times n$ matrix:

$$\hat{\mathrm{X}}(\hat{\mathbf{p}};\alpha) = \begin{bmatrix} \hat{x}_1(\hat{\mathbf{p}};\alpha) & & 0 \\ & \ddots & \\ 0 & & \hat{x}_n(\hat{\mathbf{p}};\alpha) \end{bmatrix}, \tag{A.3}$$

and $\mathbf{c}(\alpha) = (c_1(\alpha), c_2(\alpha), .., c_n(\alpha))$.

It is well known from general equilibrium theory (Arrow and Hahn (1971)) that a sufficient condition for $\hat{\mathbf{p}}$ to be unique is that the $n \times n$ matrix $\hat{X}(\hat{\mathbf{p}}) + \hat{K}(\hat{\mathbf{p}})$ be *positive definite*, where $\hat{K}(\hat{\mathbf{p}})$ is the $n \times n$ matrix whose elements are $\hat{k}_{ij} = \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_i - c_i(\alpha))s_{ij}(\hat{\mathbf{p}};\alpha)dF(\alpha)$, $s_{ij}(\hat{\mathbf{p}};\alpha) = \dfrac{\partial \hat{x}_i(\hat{\mathbf{p}};\alpha)}{\partial p_j}$ $i, j = 1, 2, .., n$.

Furthermore, if the price of each good is postulated to change in opposite direction to the sign of the profits of this good, then this condition also implies that price dynamics are globally stable, converging to the unique $\hat{\mathbf{p}}$.

Intuitively, as seen from (A.1), an upward perturbation of p_1 raises π_1 iff $\hat{x}_1 + \int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_1 - c_1)s_{11}dF(\alpha) > 0$, leading to a decrease in p_1. A simultaneous upward perturbation of p_1 and p_2 raises π_1, and π_2 the 2×2 upper principal minor of Δ is positive, and so on. Convexity of profit functions is the standard assumption in general equilibrium theory.

Appendix B.

Proof of Proposition 1.

Assume that $\varepsilon_{ji}(\hat{\mathbf{q}};\alpha) = \dfrac{\hat{q}_i s_{ji}(\mathbf{q};\alpha)}{\hat{x}_j(\mathbf{q};\alpha)}$ increases with α. Since in equilibrium

$$\int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_j - c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)dF(\alpha) = 0 \tag{B.1}$$

and, by assumption, $c_j(\alpha)$ increases with α, $\hat{p}_j - c_j(\alpha)$ changes sign once over $(\underline{\alpha}, \bar{\alpha})$, say at $\tilde{\alpha}$:

$$(\hat{p}_j - c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha) \gtreqless 0 \text{ as } \alpha \lesseqgtr \tilde{\alpha} \tag{B.2}$$

Hence,

$$(\hat{p}_j - c_j(\alpha))s_{ji}(\hat{\mathbf{q}};\alpha) < \frac{\varepsilon_{ji}(\hat{\mathbf{q}};\tilde{\alpha})}{\hat{q}_i}(\hat{p}_j - c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha) \tag{B.3}$$

for all $\alpha\varepsilon[\underline{\alpha}, \bar{\alpha}]$. Integrating on both sides of (B.3), using (B.1),

$$\int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_j - c_j(\alpha))s_{ji}(\alpha)dF(\alpha) < \frac{\varepsilon_{ji}(\hat{\mathbf{q}};\tilde{\alpha})}{\hat{q}_i}\int_{\underline{\alpha}}^{\bar{\alpha}} (\hat{p}_j - c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)dF(\alpha) = 0 \tag{B.4}$$

The inequality in (B.4) is reversed when $\varepsilon_{ji}(\hat{\mathbf{q}};\alpha)$ decreases with α.

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