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**WHEN SHOULD YOU STOP AND
WHAT DO YOU GET?
SOME SECRETARY PROBLEMS**

by

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Discussion Paper # 407 October 2005

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WHEN SHOULD YOU STOP AND WHAT DO YOU GET? SOME SECRETARY PROBLEMS

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ABSTRACT. A version of a secretary problem is considered: Let X_j, $j = 1, \dots, n$ be i.i.d. random variables. Like in the classical secretary problem the optimal stopper only observes $Y_j = 1$, if X_j is a (relative) record, and $Y_j = 0$, otherwise. The actual X_j-values are not revealed. The goal is to maximize the expected X-value at which one stops. We describe the structure of the optimal stopping rule, its asymptotic properties and the asymptotic expected reward. Three different families of distributions of X are considered, belonging to the three different domains of attraction of the maximum. It is shown that both the time of stopping, as well as the expected reward are strongly distribution dependent. The last section discusses an 'inverse' of 'Robbins' Problem'.

1. INTRODUCTION

Let $X_1, \dots, X_n$ be continuous i.i.d. random variables with known distribution. In the present paper we consider stopping problems where the decision when to stop must be based only on the relative ranks of the Xs, while the payoff is the (unobservable) X_j, if one stops at time j, $j = 1, \dots, n$. Except for the last section, we assume that one only knows whether the present observation, X_j, is the largest observed so far, in which case $Y_j = 1$, or not, in which case $Y_j = 0$. Two assumptions are made throughout:

(i) The horizon, n, is known.
(ii) One item must be chosen.

Assumption (ii) implies that if one has not stopped before time n, she must stop at time n.

Probably the best known optimal stopping problem is the Classical Secretary Problem (see e.g., Gilbert and Mosteller (1966)), which is based on the following additional assumptions

(a1) Only relative records ($Y_j = 1$) or nonrecords ($Y_j = 0$) are observed at time $j, j = 1, \dots, n$.
(b1) The goal is to maximize the probability of choosing the overall best item.

2000 *Mathematics Subject Classification.* Primary 60G40.

Key words and phrases. Domain of attraction, relative record, relative rank, Secretary Problem, stopping rule, stopping value.

Research supported by The Israel Science Foundation, (grant no. 467/04).

The solution to this problem is well known: When $n \to \infty$ one should let $e^{-1}n$ items pass, and choose the first j thereafter for which $Y_j = 1$. (If none exists, stop at time n.) The probability of choosing the overall best item tends to e^{-1} as $n \to \infty$.

Recently Bearden (2006) considered the following interesting twist of the above problem: (a1) remains unchanged but (b1) is replaced by the different goal

(b2) The goal is to maximize the X-value of the item chosen.

Bearden (2006) considers this problem when the X_js are i.i.d. uniformly distributed on $[0,1]$. He shows that for every $n \geq 1$ the optimal strategy is: Let $c-1$ items pass, and stop with the first $j \geq c$ for which $Y_j = 1$. (If none exists, stop at time n). The optimal value of c is either $\lfloor\sqrt{n}\rfloor$ or $\lceil\sqrt{n}\,\rceil$. Bearden refers to empirical experiments with subjects confronted with the classical secretary problem. These exhibit a tendency to stop too soon. He wonders, therefore, whether the reason might be that the subjects of the experiment have (b2) rather than (b1), in mind.

Bearden's result depends on the fact that the X_js are uniformly distributed. Hence the title of the present note: "When should you stop and what do you get?" Below we consider this problem when the i.i.d. X_js come from three different families of distributions, belonging to the three different domains of attraction for the maximum. We show that the optimal number of items one should skip depends heavily on the distribution. We also compute the asymptotic expected return, when using the optimal rule, and compare this to the asymptotic optimal expected return attainable, had the X_js themselves been sequentially observable. The last section discusses the optimal stopping problem, where stopping can be based on the sequentially observed relative ranks of the X_js and the payoff is the X_j chosen, for the case where the X_js are i.i.d. uniformly distributed.

2. General description of the optimal rule, and payoff

Let $M_j = \max\{X_1, \ldots, X_j\}$. Since $\{M_j\}$ is a monotone nondecreasing sequence, it is clear that if for some value m you should stop if $Y_m = 1$, then, if $Y_m = 0$ you should stop for the smallest $j > m$ for which $Y_j = 1$, (if such a j exists). (The reason one should never stop for $Y_j = 0$ and $j < n$ is that $E[X_j|Y_j = 0] < EX_n$, thus it is better to stop at n.) Hence, when seeking an optimal rule for n fixed, we only need to search among the class

$$t_n(k) = \min\{j > k : Y_j = 1\} \wedge n, \qquad k = 0, \ldots, n-1. \tag{1}$$

Let $V_n(k)$ be the expected return when using $t_n(k)$.

Then it is easily seen that $V_n(0) = EX$ and for $k = 1, \cdots, n-1$

$$\begin{aligned} V_n(k) &= \sum_{j=k+1}^{n-1} EM_j \frac{1}{j} \prod_{i=k+1}^{j-1} \left(\frac{i-1}{i}\right) + \prod_{i=k+1}^{n-1} \left(\frac{i-1}{i}\right) EX_n \\ &= k \left[\sum_{j=k+1}^{n-1} EM_j/j(j-1) + EX/(n-1)\right] \end{aligned} \tag{2}$$

(where the last term is due to the possibility of being forced to stop at time n).

Thus the optimal rule and value can be obtained by maximizing $V_n(k)$ over k. Denote

$$V_n = \max_{k=0,\ldots,n-1} V_n(k) \text{ and } k(n) = \arg \max_{k=0,\ldots,n-1} V_n(k). \tag{3}$$

Below we consider the three following classes of distributions, each of which belongs to a different domain of attraction of the extremal distribution.

(A) The family of Pareto distributions, with $F(x) = [1 - x^{-\alpha}]I(x \geq 1)$ where $I(A)$ is the indicator function. We consider $\alpha > 1$ only, as otherwise $EX = \infty$ and the problem is trivial.

(B) The family of Beta distributions $F(x) = [1-(1-x)^{\alpha}]I(0 \leq x \leq 1) + I(x > 1)$, where $\alpha > 0$. (For $\alpha = 1$ this yields the uniform distribution.)

(C) The exponential distribution, with $F(x) = [1 - e^{-x}]I(x \geq 0)$.

We consider asymptotic results only. It is clear that as $n \to \infty$ the optimal k-value also tends to infinity. It is quite well known (see e.g. Resnick (1987), Chapter 2.1) that

$$\begin{aligned} &\text{for (A)} \quad \lim_{j\to\infty} j^{-1/\alpha} EM_j = \Gamma(1 - 1/\alpha) \\ &\text{for (B)} \quad \lim_{j\to\infty} j^{1/\alpha}[1 - EM_j] = \Gamma(1 + 1/\alpha) \\ &\text{for (C)} \quad \lim_{j\to\infty} EM_j - \log j = \gamma. \end{aligned} \tag{4}$$

Here Γ is the gamma function and $\gamma \sim .5772\ldots$ is Euler's constant.

3. Asymptotic results for case (A)

We shall show

Assertion 1. *For family (A), and any $\alpha > 1$*

$$\lim_{n\to\infty} k(n)/n = \alpha^{-\alpha/(\alpha-1)} \tag{5}$$

and

$$\lim_{n\to\infty} n^{-1/\alpha} V_n = \Gamma(1 - 1/\alpha)\alpha^{-\frac{1}{\alpha-1}}. \tag{6}$$

Proof. From (4A) it follows that we can write, for all $j > k$ sufficiently large $EM_j = (\Gamma(1 - 1/\alpha) + \epsilon_j)j^{1/\alpha}$, where $|\epsilon_j| < \epsilon$, and $\epsilon \to 0$ as $k \to \infty$. Substituting this into (2) yields

$$
\begin{aligned}
&(\Gamma(1-1/\alpha)-\epsilon)k\sum_{j=k+1}^{n-1}(j-1)^{-1}j^{-(1-1/\alpha)}+k\alpha/[(\alpha-1)(n-1)]<V_n(k)\\
&<(\Gamma(1-1/\alpha)+\epsilon)k\sum_{j=k+1}^{n-1}(j-1)^{-1}j^{-(1-1/\alpha)}+k\alpha/[(\alpha-1)(n-1)].
\end{aligned}
\tag{7}
$$

Since $\epsilon \to 0$ as $k, n \to \infty$, for asymptotic results it suffices to analyze the right hand side, setting $\epsilon = 0$, only. The sum $S = \sum_{j=k+1}^{n-1} (j-1)^{-1} j^{(1-1/\alpha)}$ of (7) satisfies

$$\int_{k+1}^{n} \frac{dx}{x^{2-1/\alpha}} < S < \int_{k}^{n-1} \frac{dx}{(x-1)^{2-1/\alpha}}$$

and since there is no need to asymptotically distinguish between $k, k-1, k+1$ etc., nor between $n, n-1, n+1$ etc. we shall replace S in (7) by $\alpha[k^{-(1-1/\alpha)} - n^{-(1-1/\alpha)}]/(\alpha - 1)$ and write

$$V_n(k) \approx \Gamma(1-1/\alpha)\alpha k[k^{-(1-1/\alpha)} - n^{-(1-1/\alpha)}]/(\alpha-1) + k\alpha/(\alpha-1)(n-1) \tag{8}$$

Differentiating the right hand side of (8) with respect to k yields that the maximizing k should satisfy

$$\Gamma(1-1/\alpha)[k^{-(1-1/\alpha)} - \alpha n^{-(1-1/\alpha)}]/(\alpha-1) + \alpha/(\alpha-1)n = 0 \tag{9}$$

It is immediately seen that the last term in (9) has only a secondary effect. We therefore conclude that the optimal $k(n)$ must satisfy (5). Substituting (5) into the right hand side of (8) yields (13). $\square$

It is of interest to compare the result in (6) with the optimal asymptotic value attainable, had the actual X_j values been revealed sequentially, $j = 1, \ldots, n$ and the goal is to stop with as large an X-value as possible. This problem has been studied in detail by Kennedy and Kertz (1991), for the three domains of attraction. From their results it follows that

$$\lim_{n\to\infty} n^{-1/\alpha} W_n = [\alpha/(\alpha-1)]^{1/\alpha} \tag{10}$$

where we have denoted by W_n the optimal expected reward from the X_js themselves, in an n-horizon problem. Comparing (10) and (6) and we see that although in our present context only the location of relative records is being sequentially revealed, no rate of convergence is lost, though the constants in the right hand side of (6) are clearly smaller than those of (10). When $\alpha \to \infty$ both constants tend to 1. In Table 1 we list some values of these constants, and their ratio.

The limit in (5) is also interesting. It shows that for family (A) one should let a proportion of n pass, before considering choosing the first relative record. It is quite easily seen, that this proportion, $\alpha^{-a/(\alpha-1)}$, is monotone decreasing in α, and satisfies $\lim_{\alpha\to\infty} \alpha^{-\alpha/(\alpha-1)} = 0$ and $\lim_{\alpha\to 1} \alpha^{-\alpha/(\alpha-1)} = e^{-1}$. (The latter constant is of interest, when comparing to the classical Secretary Problem).

Remark. The classical Prophet Inequality states that for any nonnegative independent random variables with finite expectations

$$EM_n < 2 \sup_t EX_t \tag{11}$$

where the supremum is over all stopping rules which stop no later than n. When the X_js are i.i.d. the constant "2" can be replaced by an even smaller (n-dependent) constant. See e.g. Hill and Kertz (1992). Comparing (4A) and (6), it is easily seen that (11) fails here, for small values of α. This is no contradiction! The stopping rules t in (11) are based on the sigma fields created by the X_j values themselves, whereas here the stopping rules are based on the Y_js only.

4. Asymptotic results for case (B)

We use the same notation as before, making no distinction due to distribution. The proof is also parallel to that of the previous section, and we omit some explanations.

Assertion 2. *For family (B) and any $\alpha > 0$*

$$\lim_{n\to\infty} k(n)/n^{\alpha/(\alpha+1)} = (\Gamma(1+1/\alpha)/\alpha)^{\alpha/(\alpha+1)} \tag{12}$$

and

$$\lim_{n\to\infty} n^{1/(1+\alpha)}(1-V_n) = \Gamma(1+1/\alpha)^{\alpha/(\alpha+1)}\alpha^{1/(\alpha+1)} \tag{13}$$

Proof. By (4B), for all j sufficiently large $EM_j = 1 - j^{-1/\alpha}(\Gamma(1+1/\alpha)+\epsilon_j)$ where $|\epsilon_j| < \epsilon$ and $\epsilon \to 0$ as $j \to \infty$. Substituting into (2) we have, for all k sufficiently large

(14)

$$1 - (\Gamma(1+1/\alpha)+\epsilon)k \sum_{j=k+1}^{n-1} j^{-(1+1/\alpha)}(j-1)^{-1} - \frac{\alpha k}{(\alpha+1)(n-1)} < V_n(k) <$$

$$1 - (\Gamma(1+1/\alpha)-\epsilon)k \sum_{j=k+1}^{n-1} j^{-(1+1/\alpha)}(j-1)^{-1} - \frac{\alpha k}{(\alpha+1)(n-1)}$$

Approximating the sum in (14) by the integral

$$\int_k^{n-1} x^{-(2+1/\alpha)}dx = \frac{\alpha}{\alpha+1}[k^{-(1+1/\alpha)} - (n-1)^{-(1+1/\alpha)}]$$

and substituting back into (14) , one obtains

$$V_n(k) \approx 1 - \Gamma(1+1/\alpha)\alpha[k^{-1/\alpha} - k(n-1)^{-(1+1/\alpha)}]/(\alpha+1) - \frac{\alpha k}{(\alpha+1)(n-1)} \tag{15}$$

Differentiating (15) with respect to k, we see that the maximizing k should satisfy

$$\Gamma(1+1/\alpha)[k^{-(1+1/\alpha)}+\alpha(n-1)^{-(1+1/\alpha)}]/(\alpha+1)-\frac{\alpha}{(\alpha+1)(n-1)}=0 \tag{16}$$

Note that, in contrast to what happened for family (A), in (16) the last term is significant, and the smaller order term which we can ignore is $(n-1)^{-(1+1/\alpha)}$. Solving for k yields (12). (Note that for $\alpha = 1$ (12) reads $k(n)/\sqrt{n} \to 1$, in agreement with Bearden's (2006) exact result for the uniform distribution which is a particular case of (B).) Substituting the value for $k(n)$ of (12) back into (15) yields (13). $\square$

It is of interest to note that when the values of the X_js themselves are observed sequentially $j = 1, \dots, n$, the optimal return, W_n, for family (B) (see Kennedy and Kertz (1991)) satisfies

$$\lim n^{1/\alpha}(1-W_n)=(1+1/\alpha)^{1/\alpha} \tag{17}$$

Comparing (17) and (13), we see that unlike for family (A) for family (B) the rate of convergence of V_n to 1 in (13) is slower than that of W_n to 1 in (17) .

5. Asymptotic results for case (C)

Substituting $EM_j = \log j + (\gamma + \epsilon_j)$ into (2), and noting that for sufficiently large k and $j > k$ one has $|\epsilon_j| < \epsilon$ and $\epsilon \to 0$ as $k \to \infty$, we get

$$\begin{aligned} &k[\sum_{j=k+1}^{n-1}(\log j+(\gamma-\epsilon)/j(j-1)+1/(n-1)]<V_n(k)\\ &<k[\sum_{j=k+1}^{n-1}(\log j+(\gamma+\epsilon))/j(j-1)+1/(n-1)] \end{aligned} \tag{18}$$

Approximate the sum in (18) by

$$\int_k^{n-1}[\frac{\log x}{x^2}+\frac{\gamma}{x^2}]dx=(\log k+(1+\gamma))/k-(\log(n-1)+(1+\gamma))/(n-1) \tag{19}$$

Thus

$$V_n(k)\approx\log k+(1+\gamma)-k(\log(n-1)+\gamma)/(n-1) \tag{20}$$

Differentiating (20) with respect to k yields

Assertion 3. *For distribution (C)*

$$\lim_{n\to\infty} k(n)\log n/n=1 \tag{21}$$

$$\lim_{n\to\infty}(V_n-\log n+\log\log n)=\gamma \tag{22}$$

The value in (22) is obtained by substituting (21) into (20). The value W_n, if the X_js themselves were observable, satisfies

$$\lim(W_n - \log n) = 0 \tag{23}$$

Also here, V_n converges at a slightly slower rate than W_n.

6. A different goal

Suppose now that goal (b2) is replaced by

(b3): The goal is to maximize the X-value chosen, but only if that is the maximal item. Otherwise the payoff is 0.

The assumption (a1) remains unchanged. Let t_n^* be the rule which maximizes the probability of choosing the best in the Classical Secretary Problem, with horizon n, and let $P(n)$ denote the probability that the best is chosen.

Assertion 4. Let $X_1, \cdots, X_n$ be any non-negative i.i.d. random variables with continuous distribution. Under (a1) and (b3) the optimal rule is t_n^* of the Classical Secretary Problem and the optimal expected payoff is $EM_nP(n)$.

Proof. An intuitive proof is the following. Since the expected value of M_n does not depend on its location, the goal is equivalent to that of finding that location, i.e. of choosing the best. A more formal proof follows. The argument in Section 2 shows that one needs only consider rules $t_n(k)$ of (1). Suppose $Y_j = 1$ and one stops. The payoff then is $X_j \prod_{i=j+1}^{n} I(Y_i = 0)$, where, conditional on $Y_j = 1$, X_j has distribution $F_{(j)}$ of the maximum of j i.i.d. random variables with distribution F, i.e. $dF_j(x) = jF(x)^{j-1}dF(x)$. Now $E\{\prod_{i=j+1}^{n} I(Y_i = 0)|X_j, Y_j = 1\} = F(X_j)^{n-j}$. Thus $E\{X_j \prod_{i=j+1}^{n} I(Y_j = 0)|Y_j = 1)\} = \int_0^\infty xjF(x)^{n-1}dF(x) = \frac{j}{n}EM_n$. For $k \geq 1$ let $S_n(k)$ denote the expected payoff when using rule $t_n(k)$. Then $S_n(k) = \frac{EM_n}{n} \sum_{j=k+1}^{n} j \cdot \frac{1}{j} \prod_{i=k+1}^{j-1} \left(\frac{i-1}{i}\right) = \frac{EM_n}{n} \sum_{j=k+1}^{n} \frac{k}{j-1}$. Thus the maximizing k must satisfy $k \sum_{j=k+1}^{n} 1/(j-1) \geq (k+1) \sum_{j=k+2}^{n} 1/(j-1)$, which is equivalent to $\min\{k : \sum_{j=k+1}^{n} 1/(j-1) \leq 1\}$, which is exactly t_n^*. Also $(k/n) \sum_{j=k+1}^{n} 1/(j-1)$ is the probability of choosing the best, with rule $t_n(k)$. □

Remark. If the stopping rule is based on the X-values themselves, the optimal rule for goal (b3) depends on the distribution of the Xs. We hope to come back to this, in a future paper.

7. The "inverse Robbins' problem"

Chow et al (1964) considered a different version of the Secretary Problem. In their model, (a1) is replaced by

(a2) The (sequential) observations on which the stopping rule is based, are the *relative ranks* RR_j of the X_js, where $RR_j = \sum_{i=1}^{j} I\{X_i \geq X_j\}$.

(Thus $RR_j = 1$ if the jth observation is the largest, $= 2$ if it is the second largest, etc. among $X_1, \dots, X_j$.) The goal (b1) is replaced by

(b4) The goal is to minimize the expected absolute rank of the object chosen.

Let $R_j^{(n)}$ be the (absolute) rank of X_j, i.e. $R_j^{(n)} = \sum_{i=1}^{n} I\{X_i \geq X_j\}$, and denote by T_R^n the set of stopping rules based on the relative ranks, which stop by time n. Chow et al prove

$$\lim_{n\to\infty} [\min_{t \in T_R^n} ER_t^{(n)}] = \prod_{k=1}^{\infty} \left(\frac{k+2}{k} \right)^{1/(k+1)} = 3.8695\dots \tag{24}$$

They also describe the structure and asymptotic structure of the optimal stopping rule.

"Robbins' Problem" can be described as follows. (b4) is as described above, but (a1) and (a2) are replaced by

(a3) The (sequential) observations on which stopping must be based, are the i.i.d. X-values themselves. These can, without loss of generality, be taken as $U(0,1)$.

This problem is as yet unsolved, in spite of several attempts to solve it. A recent review paper is Bruss (2005). Let Q_n denote the expected optimal return for horizon n. It is known that $\lim_{n\to\infty} Q_n$ exists and is smaller than 2.3267, but it is not known whether $\lim Q_n$ is greater or smaller than 2. The value "2" is of interest, since if (b4) is replaced by

(b5) The goal is to minimize the X-value chosen,

it follows readily from (17) with $\alpha = 1$, that for the uniform distribution

$$\lim_{n\to\infty} n[\min_{t \in T_X^n} EX_t] = 2 \tag{25}$$

where T_X^n denotes the set of stopping rules based on the X-values, which stop by time n.

(The reason we are interested in minimization rather than maximization, i.e. we replace (b2) by (b5), is to be in line with the goal in (b4). The need to multiply by n is to make the X-values comparable to the absolute ranks, but this need is also clearly seen from (17).)

The "inverse Robbins' problem" is where the role of the observations and the ranks have been reversed. Thus the goal is to minimize the expected value of the X at which you stop, but the decision as to when to stop may depend on the relative ranks only. The assumptions are therefore (a2) and (b5) where it is assumed that the Xs are i.i.d. $U(0,1)$.

Unlike the original Robbins' Problem, the inverse Robbins' problem has a simple solution

Assertion 5. *For X_i uniformly distributed on $[0,1]$, with assumptions* (a2) *and* (b5)*,*

$$\lim_{n\to\infty} n[\min_{t\in T_R^n} EX_t] = \prod_{k=1}^{\infty}(\frac{k+2}{k})^{1/(k+1)} = 3.8695\ldots \tag{26}$$

and the optimal rule is identical to that described in Chow et al (1964).

Proof. For the uniform distribution, clearly, by considering order statistics, one has $E(X_j|RR_j = k) = \frac{k}{j+1}$, and the relative ranks are independent, thus conditioning on past and present relative ranks is equivalent to conditioning on the present relative rank only. On the other hand (see also Chow et al (1964))

$$P(R_j^{(n)} = m|RR_j = k) = \binom{m-1}{k-1}\binom{n-m}{j-k}/\binom{n}{j},\ m = k,\cdots,n$$

and thus $E(R_j^{(n)}|RR_j = k) = \frac{n+1}{j+1}k$. Therefore

$$(n+1)E(X_j|RR_j = k) \equiv E(R_j^{(n)}|RR_j = k). \tag{27}$$

and the assertion follows immediately from (24). □

Remark. It is clear that the assertion is again distribution-dependent. We have not attempted to find the asymptotic value and optimal rule for distributions other than the uniform.

Table 1: Comparison of Limiting Optimal Stopping Values for Distribution (A)

α	(6)	(10)	(6)/(10)
1.01	37.1310	96.4887	0.3848
1.05	7.7141	18.1659	0.4247
1.10	4.0505	8.8455	0.4579
1.20	2.2370	4.4510	0.5026
1.30	1.6456	3.0893	0.5327
1.40	1.3579	2.4469	0.5549
1.50	1.1906	2.0801	0.5724
1.60	1.0830	1.8460	0.5867
1.70	1.0090	1.6853	0.5987
1.80	0.9559	1.5691	0.6092
1.90	0.9163	1.4818	0.6184
2.00	0.8862	1.4142	0.6267
3.00	0.7818	1.1447	0.6830
4.00	0.7720	1.0746	0.7184
5.00	0.7786	1.0456	0.7446
6.00	0.7888	1.0309	0.7652
7.00	0.7995	1.0223	0.7821
8.00	0.8096	1.0168	0.7962
9.00	0.8189	1.0132	0.8083
10.00	0.8274	1.0106	0.8187

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