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A THEORY OF MUSICAL CHAIRS

by

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STRATEGIC MERGER WAVES: A THEORY OF MUSICAL CHAIRS

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Abstract. This paper proposes an explanation of merger waves based on the interaction between competitive pressure and irreversibility of mergers in an uncertain environment. A set of acquirers compete over time for scarce targets. At each point in time, an acquirer can either postpone a takeover attempt, or raid immediately. By postponing the takeover attempt, an acquirer may gain from more favorable future market conditions, but runs the risk of being preempted by rivals. First, a complete information model is considered, and it is shown that the above tradeoff leads to a continuum of subgame perfect equilibria in monotone strategies that are strictly Pareto ranked. All these equilibria share the feature that all acquirers rush simultaneously in merger waves. The model is then extended to a dynamic global game by introducing slightly noisy private information about merger profitability. This game is shown to have a unique Markov perfect Bayesian equilibrium in monotone strategies, and the timing of the merger wave can thus be predicted. Last, the comparative dynamics predictions of the model are related to stylized facts.

Keywords: Merger waves, preemption, dynamic global games, real options games.
JEL Classification: C73, D92, G34, L13.

1. Introduction

Mergers and acquisitions (M&A) come in waves, both economy-wide and industry-wide. During these waves, billions worth of assets change hands. In 1995 alone, the value of M&A equaled 5% of United States GDP.[1] The economic importance of mergers and acquisitions is thus hard to ignore. Consequently, a vast empirical literature has sought to uncover the forces leading to mergers.[2] The evidence suggests that macroeconomic variables play an important role in determining the timing of mergers. Specifically, merger activity is found

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[1]See Andrade and Stafford (1999).

[2]For extensive reviews of the empirical literature on merger activity see, Golbe and White (1988), Weston, Chung and Hoag (1990) and Blair and Schary (1993).

to be highly procyclical, slightly leading the business cycle. Other research has documented a positive relation between merger activity and factors such as economy-wide dispersion in Tobin's q (Jovanovic and Rousseau, 2002) and industrial production (Gort, 1969 and Mitchell and Mulherin, 1996). On the other hand, the business and popular press often stress that managers take other managers' actions into account when deciding on if and when to merge. The aim of this paper is to build a theory that can explain why mergers happen in waves, incorporating both dependence on exogenous factors, such as aggregate activity, and strategic interdependence between firms' decisions.

Merger wave theories can be categorized according to whether or not they incorporate strategic elements. I will refer to them as *strategic* and *non-strategic* theories, respectively. Strategic theories of merger waves explicitly account for the mechanism through which one merger is related to the other. For example, the industrial organization literature has focused almost exclusively on strategic interaction through the product market.[3] At the other end, non-strategic theories of merger waves emphasize the effects of exogenous factors such as deregulation, globalization or the introduction of new technologies. In this context, merger waves are characterized by the fact that it is not the merger activity of other firms *per se* that induces a firm to merge, but rather an exogenous shift in the economic environment that simultaneously makes all mergers attractive.

For example, Gort (1969) and Mitchell and Mulherin (1996) report evidence that M&A activity is significantly correlated with technological shocks and generally with disturbance to the economy or a specific industry. In line with these findings, Jovanovic and Rousseau (2002) show that bursts in merger activity may follow from technological shocks as physical assets are reallocated from less efficient targets to more efficient acquirers. In this view, a merger wave is the effect of inefficiencies caused by exogenous shifts in the economic environment. Faria (2002) presents a model in which mergers serve as a vehicle for the transfer of managerial skills (intangible assets), while Lambrecht (2001) and Morellec and Zhdanov (2003) emphasize non-strategic real options aspects of the merger decision. Last, Rhodes-Kropf and Viswanathan (2003) show that in situations with economy-wide misvaluations, targets may have a larger propensity to accept takeover offers. In all these settings, the timing of mergers is disconnected from strategic considerations.

In practice, both strategic and non-strategic elements seem to play an important role in creating merger waves. Blair and Schary (1993) discuss these issues at length in the context of the 1980's merger wave, and conclude that "...[the evidence] suggests a formal model of [merger] activity as a function of a set of macroeconomic and industry-specific conditions [...]. [Mergers are] triggered when those conditions reach some threshold point". Similarly, Mitchell and Mulherin (1996) study the influence of macroeconomic variables as well as industry specific shocks on the timing of M&A activity and conclude that "our results suggest that a fruitful research design would consider the joint effect of macroeconomic and industry-level factors in modeling the behavior of takeovers over time." Last, Cabral (2000) states that "real-world examples of [merger waves] in particular industries suggest that both

[3]Fauli-Oller (2000) builds a simple four-firm oligopoly model in which some equilibria can be characterized as merger waves. However, his results predict that merger waves should coincide with declining markets, which is in contrast to the evidence. Fridolfsson and Stennek (2002) consider a three firm oligopoly model where mergers yield synergies so large that the negative externality conferred upon non-merging firms may prompt these to merge preemptively.

exogenous and endogenous effects are present".

This calls for new theory that encompasses both features. In the present work, I propose a dynamic model of merger activity in which waves occur as an equilibrium phenomenon. An underlying economic fundamental influencing merger profitability is modeled as an exogenous stochastic process, but merger waves occur as a result of strategic interaction. A strategic merger wave in the current setting will be interpreted as a situation in which the exogenous economic conditions prompting a firm to seek a merger vary discontinuously with the merger activity of other firms.

The model builds on three simple ingredients. First, I pose that there is relative *scarcity* of potential desirable targets. This is a plausible assumption, given that there are often multiple suitors for specific targets. Empirical studies, e.g. Bradley, Desai and Kim (1988) and De, Fedenia and Triantis (1996), show that competition between multiple bidders for a single target hurts bidders. Revealed preferences thus suggest that suitable alternative targets cannot be abundant. As a practical matter, there is usually no problem in distinguishing between potential targets and acquirers, where the identities of the acquirer and the target are determined by some notion of size, e.g. capacity, market share or market capitalization.[4] For example, in the world airline industry, there is a natural distinction between European and North American airlines. There is also a sense among the latter that potential European targets for takeovers or strategic alliances are scarce. Note that the interpretation of scarcity of targets need not be literal. An alternative interpretation is that the targets own or control scarce resources or assets. Such assets could be access to restricted (geographical) markets, existing costumer bases, patents, business practices or as in the airlines example, landing slots in key European hubs. Last, one may consider a target population ranked according to some quality index, such as the ease with which the target can be successfully merged with an acquiring firm. Competition would then start for the set of high quality targets, with lower quality targets being competed over in the future.

The second ingredient driving my model is the recognition that mergers, viewed as investments, are partly irreversible and that they are carried out under conditions of considerable uncertainty. This leads to a *value of delay*, i.e. there is an *options value* in waiting to acquire a target, at least over a non-trivial period of time. Viewing an acquisition as an irreversible investment (or at least partially irreversible in the short run) is plausible, and the merger decision can then reasonably be viewed as the problem of optimally exercising a real option.[5] Delaying a merger may allow firms to look for the best fit; or it may be that the returns from the merger are realized in the future (when new markets are created), whereas implementation costs are borne immediately after the merger. Also, technological progress or convergence of hitherto separate industries may make it optimal not to merge straight away. Last, waiting may be valuable in resolving uncertainty.[6]

The third ingredient of the present model is that competition for targets is imperfect. Specifically, what is ruled out is competition à la Bertrand with homogeneous products, where all rents are dissipated. If there was a perfectly functioning price mechanism, it would "punish" a surge in demand by increasing the price level accordingly. In general, imperfec-

[4] There is an extensive literature that distinguishes acquirers from targets along values of Tobin's q. See e.g. Lang, Stulz and Walkling (1989) and Servaes (1991) and references therein.

[5] For real options models with competition, see e.g. Grenadier (2000) and references therein.

[6] For a thorough exposition of real options, see e.g. Dixit and Pindyck (1994).

tions in the price mechanism can arise because of private information, target management idiosyncrasies or agency problems. The existence of white knights and the fact that target management sometimes accepts offers that are not the most attractive, suggest that this is a plausible modeling assumption. Also, the structure and circumstances of actual offers is often very complex, consisting of different means of exchange for different amounts of shares in the target company. Also, some bidders may have toeholds (i.e. initial ownership of a stake) in the target. All these features may work against perfect competition.

I first consider a complete information model where a measure of raiders compete over time for a smaller measure of targets. I show that there exists a continuum of subgame perfect equilibria. In all equilibria, *all potential acquiring firms raid the target firms simultaneously*, a feature that may be interpreted as a merger wave. The intuition for this type of equilibrium is simple. While waiting is optimal when all other firms wait, fear of being stranded without a firm to merge with can lead firms to attempt a preemptive takeover. This in turn vindicates the belief that there will be a merger wave, thus leading all firms to raid.

Although all equilibria share the same qualitative features, multiplicity is problematic since it is impossible to predict the timing of the mergers. To resolve the multiplicity, I extend the model by introducing incomplete information. This is achieved by letting acquirers receive slightly imperfect private information about the realizations of the randomly evolving economic fundamental variable. In this setting, it is shown that there exists a *unique perfect Markovian Bayesian equilibrium in monotone strategies.* The timing of the merger wave can thus be predicted, and comparative analysis performed.

The tradeoff between a value of delay and competitive considerations has previously been identified in the literature. For example, Smith and Triantis (1995) point out that "In the case of acquisitions in an environment characterized by an absence of competition, a firm may delay its decision to acquire while waiting for more resolution of uncertainty regarding market conditions and other economic factors such as interest rates. However, since competition for specific targets is often significant, firms in practice may not be able to wait indefinitely to acquire a target, but must instead react quickly at the right time." The musical chairs metaphor is routinely used in the business press to describe the environment and conditions leading to merger waves. For example, a commentator described an expected merger wave in the international industry for legal services by stating that "One of the images accountants like to use when describing the strategic thinking of law firms is that of an enormous and slightly lascivious game of musical chairs. The music is almost over and all the big Australian law firms are circling the room, trailing their coats in the direction of a handful of global law firms and the Big Five professional services firms. If the Australians are lucky, the music might last just long enough for them to attract a merger partner [...]. But if they delay, all the international merger candidates will be snapped up by the lucky few [...].[7] Clearly, practitioners and industry participants themselves view the tradeoff between preemption (strategic considerations) and exogenous economic factors (non-strategic considerations) as crucial for the decision on if and when to seek a merger. This fact lends strong support to the present modeling approach.

[7] *Law Firms Eye the Big Five for Marriage*, Financial Review, 5/9/2001. See also *A Game of Musical Chairs is Brewing*, Financial Times, 11/29/2001, *Telephone Mergers: A Heated Game of Musical Chairs*, The New York Times, 7/28/1998, *European Ad Takeover Craze Seen Crossing Atlantic*, Reuters, 9/3/2001 for similar statements.

The basic setup is described in Section 2, which also exposes some key properties of the model. In Section 3, the complete information version of the model is analyzed and merger wave equilibria are characterized. Section 4 extends the model to a dynamic global game by introducing incomplete information, and shows that when information is very precise, the timing of the merger wave can be uniquely determined. Comparative analysis is performed in Section 5, and a simple example is contained in Section 6. Section 7 offers concluding remarks and discussion. Finally, proofs of most results are relegated to the Appendix.

2. The Model

Time is discrete and indexed by the non-negative integers $t = 0, 1, 2, ...$ There is a continuum of *targets* and a continuum of *acquirers* with unit demand for a target.[8] All acquirers are risk neutral, and discount the future with the common factor $\delta \in]0, 1[$. In every period, each acquirer faces the choice between *raiding* and *waiting*. Denote by $a_t^i \in A_t^i = \{0, 1\}$ player i's action at time t, with $a_t^i = 0$ denoting waiting and $a_t^i = 1$ denoting raiding. An acquirer who waits remains inactive until the next period.

For every period $t = 0, 1, 2, ...$, let x_t and y_t denote the measures of remaining targets and acquirers respectively, and $z_t \in [0, y_t]$ the measure of acquirers who choose to raid (*raiders*). Denote by X_t, Y_t and Z_t the sets of targets, acquirers and raiders respectively. Once an acquirer decides to raid, he participates in an *allocation game* $B_t : Z_t \times X_t \times \mathbb{R} \to \mathbb{R}$ with von Neumann-Morgenstern expected payoff $R(z_t, x_t, \theta_t)$ and remains inactive in all future periods.[9] The single dimensional variable $\theta_t \in \mathbb{R}$ represents some economic fundamental that influences merger profitability.

The expected payoff $R(z_t, x_t, \theta_t)$ from participating in the allocation game is called the *raiding value*, and should be thought of as the expected value of obtaining, through some bidding process, an infinite flow of future profits. The expected *waiting value* is given by the option to raid in future periods, and thus given by the recursive expression

$$W(z_t, x_t, \theta_t) = \delta E_t \max\left[R(z_{t+1}, x_{t+1}, \theta_{t+1}), W(z_{t+1}, x_{t+1}, \theta_{t+1})\right]$$

Note that since an acquirer always has the option of waiting indefinitely, it follows that $W(z_t, x_t, \theta_t) \geq 0$ for all t. Finally, the *net waiting value* $\Delta(z_t, x_t, \theta_t)$ is defined as

$$\Delta(z_t, x_t, \theta_t) \equiv W(z_t, x_t, \theta_t) - R(z_t, x_t, \theta_t)$$

In the event that $\Delta(z_t, x_t, \theta_t) < 0$, raiding is the dominant strategy, while waiting is dominant for $\Delta(z_t, x_t, \theta_t) > 0$. The function $\Delta(z_t, x_t, \theta_t)$ is simply the options value of deferring a takeover attempt.

Next, make the following assumptions:

A1 $y_0 > x_0$.

A2 The raiding value $R(z_t, x_t, \theta_t)$ is bounded, continuous in all arguments, strictly increasing in θ_t, weakly decreasing in z_t and weakly increasing in x_t with $R(z_t, 0, \theta_t) = 0$.

[8] Note that the analysis of neither the complete nor the incomplete information games depends on the continuum player assumption. Furthermore, the results of the complete information game do not depend on symmetry between the acquirers. It is still an open question wether the results derived in the incomplete information setting generalize to acquirers with non-informational asymmetries.

[9] It is implicitly assumed that there exists a unique equilibrium in the allocation game.

A3 The process $\{\theta_t\}_{t=0}^{\infty}$ is first-order Markov such that $\theta_t \,|\theta_{t-1} \sim G$, with density function g, and for $\theta_{t-1} > \theta'_{t-1}$, $G(\theta_t \,|\theta'_{t-1}) > G(\theta_t \,|\theta_{t-1})$.
A4 The raiding value and the stochastic process $\{\theta\}_{t=0}^{\infty}$ are such that for all z_{t+1},x_{t+1}

$$R(z_t, x_t, \theta_t) - \delta E[R(z_{t+1}, x_{t+1}, \theta_{t+1})|\theta_t]$$

is strictly increasing in θ_t.
A5 If $a_t^i = 1$ then $A_s^i = \varnothing$ for all $s > t$.

Assumption A1 captures the notion that targets are scarce.[10] Assumption A2 ensures that both the raiding value and the waiting value are bounded and well behaved, and furthermore that expected payoffs from raiding are decreasing in the intensity of competition (measured either as an increase in the measure of competitors z_t or as the absolute scarcity of remaining targets x_t). Last, the value of raiding is assumed to be increasing in the economic fundamental, such that higher realizations increase the benefits of merging, controlling for the level of competition. Assumption A3 states that there is persistence in the evolution of the economic fundamental, such that a higher realization of θ_t today shifts the distribution of future realizations in the sense of first-order stochastic dominance. Assumption A4 is a joint condition on the raiding value and the stochastic process which ensures that the raiding value increases at a higher rate than the waiting value. Last, assumption A5 simply states that a merger is irreversible.

Note that assumptions A2-A5, i.e. irreversibility and persistent shocks to merger profitability yield a value of delay, a standard insight of the real options literature. The requirement that the raiding value be strictly increasing in the economic fundamental imposes restrictions on the allocation mechanism B_t. Specifically, it amounts to the assumption of market imperfections such that, controlling for the level of competition, the expected benefit from merging is non-negative for the raider.[11]

It is implicitly assumed that no raider gets rationed in the allocation game as long as $z_t < x_t$. Thus, if an acquirer decides to raid, he will either merge with a target and leave the game, or be rationed, in which case the game is over. Thus, it follows that the laws of motion for the endogenous state variables are given by

$$x_t \equiv \max\left\{x_0 - \sum_{r=0}^{t-1} z_r, 0\right\}$$
$$y_t \equiv \max\left\{y_0 - \sum_{r=0}^{t-1} z_r, y_0 - x_0\right\}$$

This couple of identities captures the main strategic element in the interaction between acquiring firms. The higher the measure of raiders in any given period, the scarcer targets become in future periods, thereby eroding the options value of waiting.

[10] The assumption that targets are scarce is also made by Klemperer (1997), Bulow, Huang and Klemperer (1999), Rhodes-Kropf and Viswanathan (2003) and is standard in the very large literature on takeover auctions. See e.g. Fishman (1989) and Burkart (1995). Conceivably, this assumption could be dispensed with by also dispensing with the assumption of unit demand. A main difference with that literature is that here, only the measure of *potential* bidders is exogenous, while the *actual* measure of bidders is endogenous.

[11] This assumption does not rule out the possibility that the target obtains most of the gains from trade. What it does rule out is perfect rent equalization.

Assume throughout that $x_t > 0$ and denote by $z^t \equiv \{z_s\}_{s=t}^{\infty}$ a sequence of current and future measures of raiders. Also, let $z^t \geq \widehat{z}^t$ if $z_s^t \geq \widehat{z}_s^t$ for all $s \geq t$. Define the following:

Definition 1. *(Merger Triggers)*

Marshallian Trigger: $\underline{\theta} \equiv \inf\{\theta_t : R(z_t, x_t, \theta_t) \geq 0\}$

Strategic Trigger. $\widetilde{\theta}(z^t) \equiv \inf\{\theta_t : \Delta(z_t, x_t, \theta_t) \leq 0\}$

First-Best Trigger: $\overline{\theta}(z^t) \equiv \inf\{\theta_t : \Delta(z_t, x_t, \theta_t) \leq 0 \text{ for } z_s = 0,\ s \geq t\}$

That is, the Marshallian trigger $\underline{\theta}$ is the lowest value of θ_t at which the raiding value is non-negative. Next, the strategic trigger $\widetilde{\theta}(z^t)$ is the lowest value of θ_t such that raiding in the current period dominates waiting, given a sequence of current and future measures of raiders. Last, the first-best trigger $\overline{\theta}(z^t)$ is the lowest value of θ_t such that, even in the absence of competitive pressure, delaying a takeover one period further is not optimal.[12] The first-best trigger $\overline{\theta}(z^t)$ is simply the strategic trigger $\widetilde{\theta}(z^t)$ evaluated at z^t with $z_s = 0$, $s \geq t$. Note that $\widetilde{\theta}(z^t) \in [\underline{\theta}, \overline{\theta}(z^t)]$ and that $\widetilde{\theta}(z^t) \rightarrow \overline{\theta}(z^t)$ as $x_t \rightarrow y_t$.

The following results simply state that the above merger triggers exist and are unique, and gives some important properties they possess:

Lemma 2. *(Single Crossing in θ_t). For any sequence z^t there exists a unique $\widetilde{\theta}(z^t) \in [\underline{\theta}, \infty[$ such that $\Delta(z_t, x_t, \theta_t) > 0$ for $\theta_t < \widetilde{\theta}(z^t)$, $\Delta(z_t, x_t, \theta_t) = 0$ for $\theta_t = \widetilde{\theta}(z^t)$ and $\Delta(z_t, x_t, \theta_t) < 0$ for $\theta_t > \widetilde{\theta}(z^t)$. Furthermore, $\widetilde{\theta}(z^t)$ is weakly increasing in x_t and weakly decreasing in z_t and z^t.*

Proof: See Appendix A ■

Lemma 3. *(Single Crossing in z_t). For any sequence z^{t+1} and $\theta_t \in [\underline{\theta}, \widetilde{\theta}(z^{t+1})]$ there exists a unique $z_t^* \in [x_t, y_t]$ such that $\Delta(z_t, x_t, \theta_t) > 0$ for $z_t < z_t^*$, $\Delta(z_t, x_t, \theta_t) = 0$ for $z_t = z_t^*$ and $\Delta(z_t, x_t, \theta_t) < 0$ for $z_t > z_t^*$. Furthermore, z_t^* is weakly increasing in x_t and weakly decreasing in θ_t.*[13]

Proof: See Appendix A ■

These results have a straightforward interpretation. First, given any level of future and present competition, the expected value for an acquirer contemplating wether to raid or wait increases in the economic fundamental. In fact, both the value of raiding and waiting increase. But as the economic fundamental increases, the options value of delay is eroded, and ultimately the raiding value overtakes the value of waiting. Similarly, given a level of the economic fundamental, an increase in the measure of raiders reduces the measure of future targets, thereby increasing the opportunity cost of postponing a merger.

In fact, the interval $[\underline{\theta}, \overline{\theta}(z^t)]$ is a hysteresis band, i.e. a range for the economic fundamental in which the acquirers are characterized by inertia. That is, within this band, only a

[12] Although not explicit in the adopted notation, $\overline{\theta}$ at time t may depend on the state variable x_t, in which case a higher stock of targets would increase the first-best trigger.

[13] If $[\underline{\theta}, \widetilde{\theta}(z^{t+1})] = \emptyset$ for all z^{t+1} then for $\theta_t \geq \underline{\theta}$, $\Delta(z_t, x_t, \theta_t) \leq 0$ for all $z_t \in [0, y_t]$. It is thus assumed throughout that $[0, \widetilde{\theta}(z^{t+1})] \neq \emptyset$ for some z^{t+1}.

strategic response to competitive pressure will prompt an acquirer to raid. Outside the hysteresis band, strategic considerations play no role. The intuition derives from the interaction between irreversibility and uncertainty. First note that in the current analysis, the value of raiding is interpreted as a flow, which is a function not only of the current realization, but also of the future evolution of the economic fundamental. In other words, the value of being merged remains subject to random fluctuations in the economic environment. Since an acquisition is irreversible, the acquirer must be confident that the value of being merged is not likely to disappear or become negative. In order to make it worthwhile to risk being stuck with a loss making merger, the acquirer will demand a "premium" above the level of the economic fundamental which makes him break even. This creates the hysteresis band. Competition on the other hand, works in the opposite direction. Since target scarcity must increase over time, competition decreases the value of delay, thereby effectively narrowing the region of inertia. In other words, were the takeover decision fully reversible, acquirers would raid at the point at which they break even, i.e. when the fundamental reaches the Marshallian trigger, and simply undo the merger should future conditions render it unprofitable. With irreversibility however, there is an options value of delaying the takeover attempt. In turn, this means that a takeover has an opportunity cost, since it effectively kills the option. For the acquirer to be willing to kill the option, he will require a return over and above the break-even point, i.e. will raid only for realizations of the economic fundamental above the Marshallian trigger. Now consider the effect competition. As competition effectively lowers the options value of delaying the takeover, the required return making an acquirer willing to raid is dissipated, tending to zero as competition becomes very strong.

In the analysis that follows, the fundamental variable θ_t will play a prominent role, and thus deserves some further comments. In keeping with the empirical observation that exogenous variables significantly influence the merger decision, θ_t is taken to be any variable determined at aggregate level, such as technological progress, interest rates, a stock market index or other macroeconomic variable.

3. The Complete Information Game

For the complete information game it is assumed that both past and current realizations of the economic fundamental θ_t are common knowledge. Let $h_t = (\theta_0, ..., \theta_{t-1}; z_0, ..., z_{t-1})$ denote history at time t and H_t the set of all possible histories at time t. In this setting, a monotone strategy is defined as follows:

Definition 4. *A monotone Markovian strategy for the complete information game is a mapping* $a : \mathbb{R}^{2t} \times \mathbb{R} \rightarrow \mathbb{R}$ *such that* $(h_t, \theta_t) \longmapsto a(h_t, \theta_t) \equiv k_t$.

Thus, for a history h_t and current fundamental θ_t, a strategy picks a real number k_t with the interpretation that an acquirer raids whenever $\theta_t \geq k_t$ and waits whenever $\theta_t < k_t$. Given a strategy k_t, the chosen action a_t will thus be

$$a_{k_t}(\theta_t) = \begin{cases} 1 & \text{for} \quad \theta_t \geq k_t \\ 0 & \text{for} \quad \theta_t < k_t \end{cases}$$

where 1 stands for raid and 0 stands for wait and $a_{k_t}(\theta_t)$ denotes an indicator function. With this definition in place, the following result can be stated:

Proposition 5. *(Merger Waves) For any sequence $z^{t+1}$and history $h_t \in H_t$, any cutoff $k_t \in [\underline{\theta}, \widetilde{\theta}(z^{t+1})]$ constitutes an equilibrium strategy. Furthermore, there exist no equilibria in asymmetric strategies.*

Proof: See Appendix B ■

Thus there is a continuum of equilibrium cutoffs in each period. These are strictly Pareto ranked, with higher cutoffs dominating lower ones. It follows that even the Pareto efficient subgame perfect equilibrium is strictly inefficient, as it induces firms to merge at a suboptimally low level of the economic fundamental. Note that all these equilibria are also perfect equilibria.

Corollary 6. *(Indeterminacy) A merger wave may be triggered in any period t where $\theta_t \in [\underline{\theta}(x_t, 0), \widetilde{\theta}(z^{t+1})]$. Furthermore, a merger wave may be triggered no earlier than $\underline{t} \equiv \inf\{t : \theta_t \geq \underline{\theta}\}$ and no later than $\overline{t} \equiv \inf\{t : \theta_t \geq \overline{\theta}(z^t)\}$.*

Proof: Follows immediately from Proposition 5 ■

A few comments on the interpretation of these equilibria as merger waves is warranted. In all equilibria in this version of the model, all acquirers raid simultaneously in bursts. This is not inconsistent with the evidence, as shown by Mitchell and Mulherin (1996). Their analysis suggests that the characteristic smooth wave pattern in aggregate M&A time series is caused by much more intense bursts in activity at the industry level that peak at different points in time. They state that "[the] clustering [in time] of takeover activity by industry contrasts with the more evenly distributed takeover and restructuring activity for the full sample [of all industries]."

As the corollary suggests, there is no clear way in which to determine the equilibrium outcome. At this stage, the only thing that can be said is that $\underline{t}$ is the earliest time and $\overline{t}$ the very latest time at which a rush can occur, where $\overline{t}$ is just the first point in time where the fundamental is above the first-best trigger and $\underline{t}$ is the first point in time where the fundamental is above the Marshallian trigger. Actually, this claim is weaker than necessary, for there is indeed no equilibrium with acquirers rushing at time $\overline{t}$. Postponing a takeover this long is inconsistent with the pressure caused by any reasonable threat of preemption. An immediate result is thus that in all equilibria, a merger wave will happen in finite time with probability one. In other words, there is no equilibrium in which all acquirers postpone their takeovers indefinitely.

It has been argued (see e.g. Fudenberg and Tirole, 1985 and a related discussion in Carlsson and van Damme, 1993) that Pareto optimal equilibria should take precedence over other equilibria on the grounds that players "should be" able to coordinate on good outcomes, and that these thus be adequate predictions of equilibrium play. Be that as it may, the fact remains that without some explicit theory to guide the selection of equilibrium, model-based predictions are a delicate matter. As is well known, settings with strategic complementarities and multiple equilibria leave ample room for self-fulfilling beliefs and payoff-irrelevant sunspots, and is thus an inadequate vehicle for doing comparative statics/dynamics exercises. The problem is that without any knowledge of how equilibrium is reached, comparative analysis depends on which equilibrium one takes as the benchmark, thereby inviting questions about the robustness of its predictions. In essence, what creates multiplicity are the assumptions of complete information and common knowledge. These assumptions imply that acquirers are perfectly able to predict rivals' behavior in equilibrium.

In practice, these assumptions seem hard to justify, and one should in general expect at least some degree of informational differentiation. Therefore, I will now enrich the model by assuming incomplete information. This assumption will have radical implications for the equilibrium set. Namely, it will be shown that once incomplete information is introduced, a unique Markovian perfect Bayesian equilibrium in monotone strategies exists. The analysis exploits results developed by Morris and Shin (2003) for static games. Their work also contains an excellent discussion of equilibrium multiplicity in models with strategic complementarity and it's relation to higher order beliefs.

4. The Incomplete Information Game

The model is now extended to a dynamic global game by assuming that the realization of the fundamental variable θ_t is no longer common knowledge at time t. Recall that $G(\theta_t\,|\theta_{t-1})$ denotes the distribution of θ_t, conditional on past information, and denote by g the corresponding probability density function. The next step is to specify the information technology, which is characterized by the following assumptions.

A6 At time t, acquirer i receives signal $s_{it} = \theta_t + \sigma\varepsilon_{it}$ with $\varepsilon_{it} \sim F$ (density f) identically and independently distributed over t and across i.

A7 For $a > b$, $f(a-\theta)/f(b-\theta)$ is increasing in θ.

In Assumption A6, the scalar $\sigma > 0$ measures the precision of the signal. This assumption means that in each period, acquirers receive information on the current realization of the economic fundamental. Assumption A7 states that the distribution of noise F satisfies the monotone likelihood ratio property, i.e. an increase in any signal shifts the distribution of other signals in the sense of first-order stochastic dominance. This means that an acquirer who receives a high signal assigns a large probability to other acquirers receiving high signals too. This fact is important, as signals are not only used to estimate θ_t, but also to make inferences about other acquirers' signals. The monotone likelihood ratio property is implied by the assumption that the variables θ_t and $\{s_{it}\}_{i\in Y_t}$ are affiliated.

In this setting, define the following:

Definition 7. *A monotone Markovian strategy for the incomplete information game is a mapping $a : \mathbb{R}^{2t} \times \mathbb{R} \to \mathbb{R}$ such that $(h_t, s_t) \longmapsto a(h_t, s_t) \equiv k_t$.*

Given a strategy k_t the chosen action a_t will thus be given by

$$a_{k_t}(s_t) = \begin{cases} 1 & \text{for} \quad s_t \geq k_t \\ 0 & \text{for} \quad s_t < k_t \end{cases}$$

where again 1 denotes *raid* and 0 denotes *wait* and $a_{k_t}(s_t)$ denotes an indicator function. Thus, the choice variable is the cutoff level k_t. Given these strategies, the measure of raiders for given cutoff k_t is determined by the distribution of signals. But given θ_t, the distribution of a signal s is given by $F\left(\frac{s-\theta_t}{\sigma}\right)$. One can use this to express the measure of raiders as

$$z_t = y_t \left[1 - F\left(\frac{k_t - \theta_t}{\sigma}\right)\right]$$

Since the game is dynamic, the optimal action at any given point in time will depend on competitors' play in subsequent periods. Thus, players must forecast other players' actions, which in turn implies that players must forecast other players' forecasts.

It is important to realize that history only serves to the extent that it yields a prior belief $G(\theta_t \,|\theta_{t-1})$ on θ_t. Past actions have no useful informational content, and only feed through to the current decisions through their influence on the state variables x_t and y_t. Denote by $\Delta_\sigma(s_t, k_t)$ the expected net waiting value when signal s_t is observed, and all other acquirers use cutoffs k_t.

The following result shows that under the maintained assumptions, when information becomes very precise, there exists a unique Markovian perfect Bayesian equilibrium in monotone strategies.

Proposition 8. *(Uniqueness in Infinite Horizon Game) As $\sigma \to 0$, for each sequence of realizations $\{\theta_t\}_{t=0}^{\infty}$, there exists a unique sequence of cutoffs signals $\{s_t^*\}_{t=0}^{\infty}$ such that for any history $h_t \in H_t$: $\Delta_\sigma(s_t^*, s_t^*) = 0$, $\Delta_\sigma(s_t, s_t^*) > 0$ for $s_t < s_t^*$ and $\Delta_\sigma(s_t, s_t^*) < 0$ for $s_t > s_t^*$.*

Proof: See Appendix C ■

The idea of the proof is to first break up the dynamic game into a sequence of properly constructed static games. These static games simply correspond to the stages of the dynamic game, with the "waiting value" in the static game corresponding to the equilibrium continuation value in the dynamic game. Each of these static games are amenable to the techniques developed by Morris and Shin (2003). In order to follow this approach, the equilibrium continuation values must be well defined as a function of current data. That this is indeed the case is shown by actually constructing the equilibrium continuation value by induction and showing that they are well defined. One may then focus on a sequence of static games. The key to the proof of uniqueness at each stage is to note that as noise becomes negligible, an acquirer cannot, based on his signal, make inferences about his signal's relative rank among the signals of the population of acquirers. This implies that the proportion of rivals choosing different actions is a uniformly distributed random variable. But given a uniform distribution of actions, and given the single crossing property of the acquier's net waiting value in θ_t, there is a unique signal at which indifference obtains. Once the existence of a unique indifferent acquirer has been established, the last step is to verify that for signals below the cutoff, waiting is optimal, while for signals above the cutoff, raiding is optimal. In the complete information game, it was proved that there are no equilibria in asymmetric strategies. This result carries over to the incomplete information game and thus one can restrict attention to symmetric cutoffs k_t.

More formally, the proof of Proposition 8 uses the following four-step procedure. First, the infinite horizon game is truncated to obtain a finite horizon game, denoted by $\Gamma(T) \equiv \{\Gamma_t\}_{t=0}^{T}$. Second, due to the recursive structure of $\Gamma(T)$, it is possible, for all t, to associate Γ_t with a simplified (*associated*) static game Γ_t^*, for which uniqueness can be shown by using the techniques developed for static games by Frankel, Morris and Pauzner (2003), and Morris and Shin (2003). This association is achieved by showing that in each period t, the function constituting the waiting value in Γ_t is well defined as a function of current information and actions. By solving the game backwards, the players are faced with an essentially static problem in each period. The third step is to show that the truncated *(underlying)* game $\Gamma(T)$ (where payoffs depend on the realization of the state and where history is informative about the distribution of this period's realization) converges uniformly to the sequence of associated games $\Gamma^*(T) \equiv \{\Gamma_t^*\}_{t=0}^T$, as noise becomes negligible. This implies that the underlying truncated game $\Gamma(T)$ has a unique perfect Bayesian equilibrium in Markovian cutoff strategies. The last step is to show that equilibria of the truncated game converge as the horizon recedes so that equilibria of $\Gamma(T)$, as $T \to \infty$, coincide with the equilibria of the infinite horizon game $\Gamma(\infty)$. Using the terminology of Morris and Shin (2003), the problem to be solved in each period satisfies *action single crossing* (established in Lemma 3), *state monotonicity* (established in Lemma 2), *limit dominance* (follows from the existence of $\underline{\theta}$ and $\overline{\theta}(z^t)$) and a *monotone likelihood ratio property* of the signal distribution (Assumption A7). Under these and some continuity conditions, Morris and Shin (2003) show that there exists a unique Bayesian equilibrium in monotone strategies.

Corollary 9. *(Unique Timing of Merger Wave) The merger wave will be triggered at time* $t^* \equiv \inf\{t : \theta_t \geq \theta_t^*\}$.

Proof: Follows immediately from Proposition 8 ■

Recall that under complete information, there was a continuum of realizations of the economic fundamental which could constitute equilibrium cutoffs. The striking result of Proposition 8 is that under incomplete but very precise information, there exists only one equilibrium cutoff θ^* (which is of course a function of the state variables).[14] The cutoff θ^* will be referred to as the *risk-dominant trigger*. When an acquirer observes a signal equal to the risk-dominant trigger, he is exactly indifferent between raiding and waiting. For higher signals, waiting is too risky; for lower signals, waiting is expected to yield higher payoffs.

Continuing the discussion of the previous section, the fact that equilibrium is unique allows acquirers to incorporate the risk of preemption properly into their calculations, without having to resort to speculation about which equilibrium will be played. Thus, the risk-dominant trigger reflects the required return making an acquirer willing to raid, correctly adjusting this return for the riskiness of delaying a takeover attempt. In the next section, comparative analysis of the equilibrium will be performed with respect to the parameters of the model. In a nutshell, this analysis amounts to tracking the effects that competition and the basic options value of delay have on the required return from a takeover.

5. Comparative Analysis

Although the model presented here is reduced form, it still allows for an identification of factors influencing the timing of mergers. Figure 1 illustrates how the different triggers are ordered and a sample path of the economic fundamental. The first variable of interest is the stock of target firms x_t. Ceteris paribus, a smaller measure of targets increases scarcity, thereby eroding the options value of delaying a takeover. The effect of lower x_t is thus to shift both the strategic and the risk-dominant triggers downwards. In the extreme case where there are very few targets, one should expect an almost immediate rush, although this may not be identified empirically as a merger wave, since it involves very few takeovers. An increase in the measure of potential acquirers y_t has the exact opposite effect as a decrease in x_t.

[14] Of course $\theta^* = \underline{\theta}$ cannot be excluded, in which case the equilibrium is degenerate.

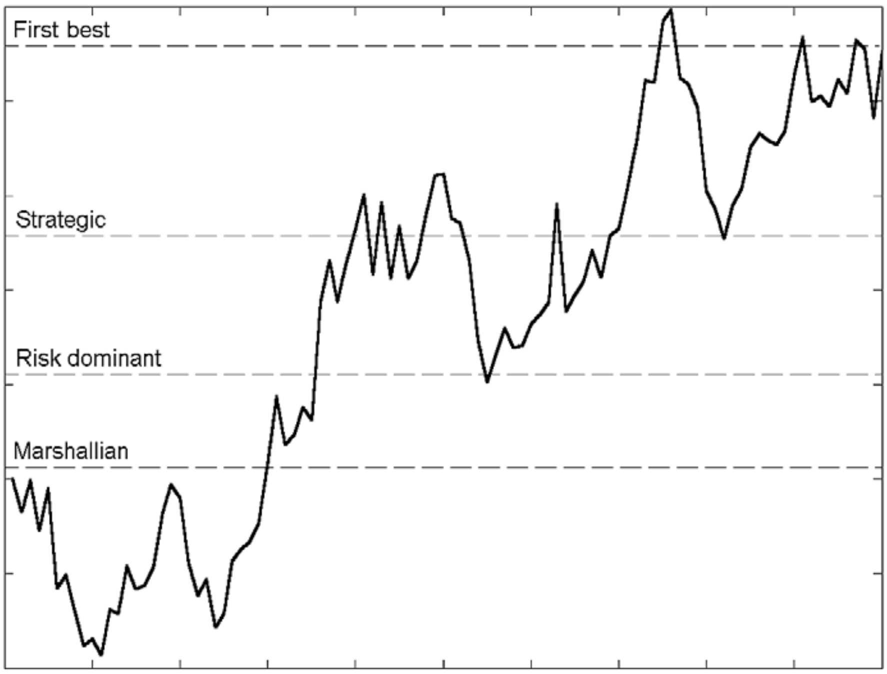

Figure 1: Triggers and sample path of θ_t.

Second, the evolution of the economic fundamental determining merger profitability has a direct implication for the expected timing of the wave. The higher the growth rate in the stochastic process, the less damaging it is for an acquirers who has already merged to be in the non-profitability region of the economic fundamental. Holding fixed the volatility of the process, a higher growth rate means that even if current conditions yield losses from being merged, the process will on expectation soon move into the profitable region. In turn, this narrows the region of inertia, leading to a decrease in the strategic triggers and shifts the risk-dominant trigger downwards. The model thus predicts that the higher the growth rate of the economic fundamental, the earlier the wave occurs. This is consistent with the stylized fact that M&A activity is particularly intense in industries with fast technological progress, such as those of information technology, telecoms and pharmaceuticals.

Next, consider the effects of the volatility of the process $\{\theta_t\}_{t=0}^{\infty}$. Recall that the options value of delaying a takeover attempt stems from the possibility of benefiting from very high future realizations, while being effectively shielded from very low realizations. This suggests that the first-best trigger $\overline{\theta}(z^t)$ is increasing in the volatility of the process. While this is certainly the case in most real options models, it does not hold a priory in the current setting without further assumptions. To obtain the result, assume that the raiding value $R(z_t, x_t, \theta_t)$

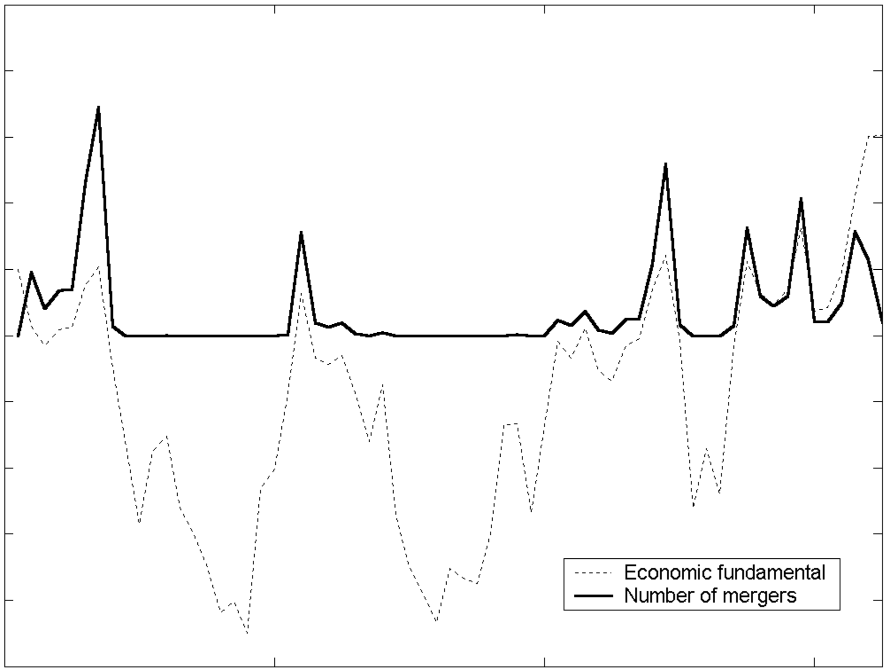

Figure 2: Simulated merger activity.

is linear in θ_t.[15] Then the expected waiting value is convex in θ_t and so is the expected net value of waiting.[16] Next, consider a mean preserving spread of the distribution $G(\theta_t\,|\theta_{t-1})$. Such a spread increases the probability mass in the tails of the distribution. Since the options value of delaying a takeover attempt is convex, standard results show that the options value is increased by the spread in the distribution. Thus, an increase in volatility increases the opportunity cost of an immediate raid, thereby making it optimal to postpone it further. Similarly, with very low volatility there is no incentive to postpone a merger (supposing of course that $\theta_t \geq \underline{\theta}$ so that the raiders break even). Without imposing linearity of the raiding value, not much can be said since the net waiting value is not in general convex. This result suggests that there may be an empirically identifiable relation between economic volatility and the intensity in M&A activity. But given the strong assumptions imposed on the model in order to obtain it, this possible relation should not be pressed too far.

While the effect of increased volatility just discussed is important, there is a second and

[15] If $R(z_t, x_t, \theta_t)$ is linear, boundedness can be achieved by restricting the state space to some bounded set $\Theta \subset \mathbb{R}$ which includes both the Marshallian and the first-best triggers. While the analysis has been carried out under the assumption that the economic fundamental takes values on the real line, parallel techniques can be applied to this reduced (but very large) state space.

[16] For the finite horizon truncation, it is also piece-wise linear.

somewhat subtler effect of increasing the level of uncertainty. Recall that knowledge of the process $\{\theta_t\}_{t=0}^{\infty}$ has two uses, namely forecasting the future evolution and for generating a prior distribution $G(\theta_t\,|\theta_{t-1})$. The latter is influenced by the volatility of the stochastic process since it determines how informative the prior distribution is. Specifically, the more volatile the process $\{\theta_t\}_{t=0}^{\infty}$ is, the less precise is public information. This has the effect of weakening the requirement of signal precision needed for uniqueness. Morris and Shin (2003) show that with general Lipschitz continuous payoff functions, normal prior and normal noise, there exists a threshold of the relative informativeness of private and public information such that uniqueness obtains whenever the relative informativeness is lower than the threshold. Specifically, uniqueness obtains when new information is much more informative than history. Their results could be adapted and applied to the present model under the assumption that $\theta_t|\theta_{t-1} \sim N(\theta_{t-1} + \mu_\theta, \sigma_\theta^2)$ and $s_{it} = \theta_t + \varepsilon_{it}$ with $\varepsilon_{it} \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. Thus, in a very volatile environment, uniqueness of a perfect Bayesian equilibrium in monotone strategies should obtain even if there is significant noise in private information. In turn, increased noise in private information increases the probability of dispersed equilibria, in which not all acquirers raid simultaneously. If a small measure of acquirers raid in a given period, the unique merger trigger in the proceeding period decreases, in turn making it more likely that some acquirers will receive signals above the new (and now lower) trigger. In this way, the wave may gain momentum and ultimately result in a rush to merge. Figure 2 illustrates simulated merger activity and a sample path of the economic fundamental with non-zero noise in private information. As is apparent from the graph, equilibria of this model generate distinct peaks in merger activity that resemble those observed in practice. For very low realizations of the fundamental, there is no merger activity. As the fundamental increases, merger activity picks up. Note that towards the end, merger activity becomes insensitive to increases in the economic fundamental. This is simply because by then, there are not many remaining targets. This is consistent with the observations of Weston, Chung and Hoag (1990), who note that "The fact that mergers peak before overall economic activity may reflect that there is at any one time a pool of firms suitable for acquisitions and, as they are acquired in a period of high merger activity, the pool is diminished and merger activity returns to a low level."

Fourth, the exact way in which the expected payoffs $R(z_t, x_t, \theta_t)$ in the allocation game B_t are determined, i.e. the way in which raiders and targets are matched and how the created surplus is divided, has implications for the timing of mergers. Many authors point to auctions theory when modeling takeover behavior, and there seems to be a consensus that most takeover contests, especially those that seek to replace inefficient management, resemble common-value auctions.[17] The comparative dynamics implications of different auction mechanisms is left for future research, but conceivably, the degree of competitiveness in some bidding games may be more sensitive than others to small changes in the measures of targets and acquirers. Leaving the exact nature of the allocation game unspecified and further assuming that the raiding value $R(z_t, x_t, \theta_t)$ is differentiable, consider the effects of increases in $R_\theta(z_t, x_t, \theta_t)$, $R_x(z_t, x_t, \theta_t)$ and $R_z(z_t, x_t, \theta_t)$, where subscripts denote partial derivatives. First, recall that the options value of postponing a merger derives in part from the variability in the raiding value brought about by the stochastic evolution of the economic fundamental.

[17] See discussion in Klemperer (1997), Cramton and Schwartz (1991) and Cramton (1998).

Thus, the larger $R_\theta(z_t, x_t, \theta_t)$ is, the more an acquirer can benefit from high realizations of the economic fundamental. This widens the hysteresis band $[\underline{\theta}, \overline{\theta}(z^t)]$. In a sense, this result is equivalent to increased volatility of the economic fundamental, and so tends to delay the merger wave. Conversely, as $R_\theta(z_t, x_t, \theta_t)$ approaches zero, the merger wave will happen at the first time where $\theta_t \geq \underline{\theta}$. An increase in $R_x(z_t, x_t, \theta_t)$ corresponds to an increase in the competitive pressure brought about by the scarcity of targets. As the scarcity is non-decreasing over time, a higher absolute value of $R_x(z_t, x_t, \theta_t)$ erodes the options value of delay, thus bringing forward the merger wave. Last, consider and increase in $R_z(z_t, x_t, \theta_t)$. Intuitively, this also corresponds to a worsening of the raiders' terms of trade vis-à-vis the targets. Again, such an increase will tend to hasten the merger wave.

The predictions of the model presented here are broadly consistent with empirical studies. Gort (1969) and Mitchell and Mulherin (1996) find evidence of considerable cross industry variation in the rate of takeover activity as a response to economic shocks. Mitchell and Mulherin (1996) cite their findings as support for the hypothesis that the shocks causing merger waves are industry-specific. My model's predictions are consistent with this view. However, the previous discussion shows that industry-specific shocks need not be the sole cause of merger waves. That follows since economy-wide shocks could have different impact on different industries if one allows for differentiation of industries by the scarcity of target firms or in the way in which raiders compete for targets. Also, even if all industries are affected qualitatively in the same way by an economy-wide shock, industry-specific factors may play an important role in how much these shocks feed through to the future prospects of that particular industry.

As mentioned above, it is a stylized fact that merger activity is highly procyclical. One explanation often proposed is that the cost of capital, proxied by some measure of real interest rates, decreases when the economy is expanding. Empirical evidence also suggests that merger waves lead the business cycle. Both these observations are consistent with the predictions of my model. If the economic fundamental is interpreted as a proxy for the state of the general economic environment, this yields procyclical merger activity in the current model. Furthermore, the merger wave is not triggered at the first-best level of the fundamental, i.e. when the fundamental is very high. Rather, mergers occur at the lower risk-dominant trigger, due to competitive pressure. Thus, although the economic fundamental may continue to rise, thereby increasing the profitability of mergers ceteris paribus, the merger wave ends when all targets have been acquired. Competitive pressure thus leads mergers to take place earlier than one should expect based solely on considerations of merger profitability, and thereby lead the business cycle. This is consistent with the observation of Weston et al. (1990), quoted in the Introduction.

Next, because the measure of raiders in any given period is endogenous and target scarcity is non-decreasing over time, the model is consistent with an interesting pattern of single-bidder versus multiple-bidder mergers over the merger wave. Specifically, single-bidder mergers should be prevalent during periods of low merger activity, when competition for targets is relatively low. During the peak of the merger wave, competition for targets is significant, and thus multiple-bidder contests should be relatively more common. In existing work on mergers and acquisitions, activity during a given period of time (quarter, year etc.) is given as stocks of single-bidder and multiple-bidder takeovers. Conceivably, it should be possible to create two separate series to study which type of contest is prevalent at a given stage of the wave. Also, such an analysis could serve as a way of discriminating between the musical chairs theory and other models of merger waves.

The evidence reported by Bradley et al. (1988) show that while the presence of multiple bidders may hurt a raider, the game played between raiders and the target is not a zero sum game. Specifically, while bidders may not gain significant returns, regardless of the presence or absence of rivals, the target benefits from the presence of multiple bidders. They suggest that more attractive targets simply attract more competition. This reading of the evidence is consistent with the results of this analysis. If indeed single-bidder contests are the norm at early stages of the merger wave, these take place under economic conditions that yield low benefits from merger. Conversely, if multiple-bidder contests are relatively more frequent at later stages of the merger wave, these take place under economic conditions yielding large benefits from mergers. While under both scenarios the target may secure himself the lion's share of the gains, leaving little or nothing to the raiders, in the latter scenario the target benefits from a stake in a more valuable merger.

6. An Example

In order to make the results less abstract, a very stylized but explicit model is now presented that fits in the general framework presented so far. Consider a setting where two separate industries believe that at some uncertain point in the future, there will be demand of products whose production requires the participation of both industries. As an example, think of providers of media and content (e.g. AOL and Time-Warner). Let T denote the point in time where this new market opens, and let V denote the profits accruing to a supplier in this market. Suppose that upon merging, the parties incur a fixed implementation cost $c > 0$. Discounted profits from a merger are thus given by $\delta^T V - c$. For $T \leq 0$ an immediate merger is optimal, while for T sufficiently large, a merger yields negative profits. The surplus created through the merger is thus strictly increasing and bounded in $\theta \equiv -T$. Last, assume that there is uncertainty about the date at which the market will open.[18]

Turning to the allocation game, assume that a target facing a single bidder engages in some bargaining over the terms of the takeover. A target facing multiple bidders picks a single bidder with probability α and engages in bilateral bargaining. As one commentator on mergers in the US IT industry put it, "[...] in the context of musical chairs, those who do not carry enough weight to have their own chair, need to choose whose lap they are going to sit upon."[19]

With probability $1 - \alpha$, the target conducts an auction. This setup is consistent with the empirical observation that both bilateral negotiations and competitive bidding take place. In the auction, assume that target i receives N_i bids. The value for the target of the offer from bidder j is given by

$$U_{ij} = b_j + v_{ij}$$

where the idiosyncratic component is random and identically and independently distributed over i, j and satisfies standard assumptions of the probit model, and b_j is bidder j's bid.[20]

[18] In this particular model, the value of a takeover (absent competitive pressure) is expected to increase strictly over time, although the fundamental may have no drift.

[19] Let's Play Musical Chairs, CBDi News, 6/13/2002.

[20] See e.g. Anderson, de Palma and Thisse (1992) for a thorough exposition.

Underlying these preferences lie non-modeled factors such as the tastes of target management, differences in corporate culture etc. Bidder j seeks to maximize

$$P_{ij}(b_j)\left[\pi(\theta) - b_j\right]$$

where

$$P_{ij}(b_j) = \frac{\exp[b_j/\mu]}{\Sigma_{k=1}^{N_i}\exp[b_k/\mu]}$$

is the probability that bidder j wins the target and $\pi(\theta) = \delta^T V - c$. In symmetric equilibrium all bids are equal, leading to

$$b^* = \pi(\theta) - \frac{\mu N_i}{1 - N_i}$$

This in turn yields equilibrium payoffs given by

$$\frac{\mu}{N_i - 1}$$

The payoff from the auction is independent of the exact value of the target, an instance of the Bertrand trap. Denote by $\pi(z_t, x_t, \theta_t)$ the expected share of the surplus obtained by a raider in the bargaining game. I explicitly let this share depend on z_t and x_t as these may influence the relative bargaining powers. The expected payoff to a raider is $\pi(z_t, x_t, \theta_t)$ for $z_t \leq x_t$. For $z_t > x_t$, expected payoffs are given by

$$\frac{x_t}{z_t}\left[\alpha\pi(z_t, x_t, \theta_t) + (1-\alpha)\left(\frac{\mu}{N_i - 1}\right)\right]$$

Summing up, the raiding value is given by

$$R(z_t, x_t, \theta_t) = I_{[0,x_t]}(z_t)\pi(z_t, x_t, \theta_t) + I_{[x_t,y_t]}(z_t)\frac{x_t}{z_t}\left[\alpha\pi(z_t, x_t, \theta_t) + (1-\alpha)\left(\frac{\mu}{N_i - 1}\right)\right]$$

where $I_{[a,b]}(z_t)$ is the indicator function. This game is simple yet realistic, and satisfies the conditions imposed on $R(z_t, x_t, \theta_t)$.

7. DISCUSSION

This paper set forth a theory to explain the occurrence of merger waves. Merger waves were derived as an equilibrium phenomenon, in a simple timing game. According to Brealy and Myers (2003), merger waves still constitute one of the ten unresolved puzzles of finance. Similarly, Weston et al. (1990) sums up the challenge to research on mergers by stating that "A complete theory of mergers should have implications on the timing of aggregate merger activity. As the matter stands, there does not exist an accepted theory which simultaneously explains motivations behind mergers, characteristics of acquiring and acquired firms, and the determinants of the levels of aggregate merger activity." This is clearly a very ambitious agenda, and probably beyond what can be addressed in any single model. The present work is an attempt to address some of these issues, leaving other for future research.

The basic forces leading to merger waves in the current work is the interaction of, on one hand, an options value of delaying a takeover, caused by irreversibility and uncertainty, and on the other hand, the risk of preemption by rivals, caused by a relative scarcity of desirable takeover targets. While the emphasis on this tradeoff clearly does not do justice to the multitude of factors influencing actual mergers, it does captures effects that are present in many mergers and merger waves. A simple extension of the model into one of incomplete information allowed the exact timing of the merger wave to be determined. Comparative dynamics analysis showed that numerous factors affect the timing of the merger wave, such as the volatility of the economic environment, the growth rate of the underlying economic forces influencing merger profitability, the competitiveness of the bidding game played by would-be acquirers, and last, the effects of the sensitivity of the benefits from merger to changes in target scarcity and changes in the economic fundamental.

The presented model has several satisfactory features. First, it builds on simple and intuitive forces which are consistent with both empirical evidence and casual observation. Furthermore, the identified tradeoff leading to waves is consistent with the thinking of practitioners, as evidenced from numerous public statements on specific merger waves. I believe this to be a desirable feature of a model built to understand actual behavior.

Importantly, the present model encompasses both dependence of the merger decision on macroeconomic variables and strategic considerations. This dual dependence has been repeatedly identified by empirical studies, but existing work has emphasized either purely strategic motives or purely non-strategic ones.

While the presented model is reduced form, the basic structure imposed on it does reflect empirically plausible forces. Furthermore, the predictions of the analysis are broadly consistent with the pattern of M&A activity which is in fact observed. First, mergers activity should pick up under beneficial economic conditions, consistent with the documented procyclicality of M&A activity. Second, because of competitive pressure, mergers happen earlier than suggested by pure profitability considerations. This suggests that merger activity should lead the business cycle. Again, this is consistent with the evidence. The model is also consistent with a particular pattern of single-bidder versus multiple-bidder takeover contests across the merger wave. In particular, it is consistent with multiple-bidder contests being concentrated at later stages of the merger wave, when competitive pressure becomes more important. This prediction does not seem to arise in existing work on merger waves. As such, this may serve to discriminate between theories in future empirical work.

There are two assumptions of the present work that it would be interesting to relax. In this model, it is assumed that the identities and measures of targets and acquirers are determined exogenously. Although some empirical work has been devoted to uncovering specific characteristics of targets and acquirers, the matter seems far from settled. The determinants of such characteristics, and especially the determinants of the distribution of such characteristics, must be analyzed within a general equilibrium framework, and is beyond the scope of the present paper.

Finally, although the present analysis has mainly focused on the interaction between acquirers, the model does not exclude the possibility that target firms play a more active role. Modeling the takeover process more explicitly seems a worthwhile exercise. Intuition suggests that targets would have an interest in delaying the takeover, thereby increasing the created surplus. This might go some way in avoiding very inefficiently timed takeovers.

APPENDICES

A. PROOF OF LEMMA 2 AND LEMMA 3

Consider the space of bounded functions Ω on $Y_t \times X_t \times \mathbb{R}$ and define the operator $M : \Omega[Y_t \times X_t \times \mathbb{R}] \to \Omega[Y_t \times X_t \times \mathbb{R}]$ by

$$MV(z_t, x_t, \theta_t) = \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t]\}$$

Fix a sequence of strategies, and by implication a sequence z^t. It will now be shown that for each t, M is a contraction mapping on the space $\Omega[Y_t \times X_t \times \mathbb{R}]$ with the sup-norm. With this norm, the space Ω is a Banach space. Let $V(z_t, x_t, \theta_t) > \widehat{V}(z_t, x_t, \theta_t)$ for all (z_t, x_t, θ_t). Then

$$\begin{aligned} MV(z_t, x_t, \theta_t) &= \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t]\} \\ &\geq \max\{R(z_t, x_t, \theta_t), \delta E[\widehat{V}(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t]\} \\ &= M\widehat{V}(z_t, x_t, \theta_t) \end{aligned}$$

Thus the mapping M satisfies monotonicity. Next, let $a > 0$. Thus

$$\begin{aligned} M[V(z_t, x_t, \theta_t) + a] &= \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) + a \,|\theta_t]\} \\ &= \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t] + \delta a\} \\ &\leq MV(z_t, x_t, \theta_t) + \delta a \end{aligned}$$

and the mapping M satisfies discounting. Therefore, by Blackwell's sufficiency conditions, M is a contraction mapping (with modulus δ) on $\Omega[Y_t \times X_t \times \mathbb{R}]$. Since by assumption $R(z_t, x_t, \theta_t)$ is bounded and continuous in all arguments, it follows by the contraction mapping theorem that there exists a unique fixed point $V(z_t, x_t, \theta_t)$ such that $MV(z_t, x_t, \theta_t) = V(z_t, x_t, \theta_t)$, and furthermore that this fixed point is bounded and continuous in (z_t, x_t, θ_t) (see e.g. Stokey and Lucas with Prescott, 1989 for details). Assume throughout that $x_t > 0$ and fix a sequence z^t.

Recall that by assumption $R(z_t, x_t, \theta_t)$ is strictly increasing in θ_t, weakly decreasing in z_t and weakly increasing in x_t. For $z_t < x_t$ it follows by the assumption of first-order stochastic dominance that the fixed point $V(z_t, x_t, \theta_t)$ is strictly increasing in θ_t. Also, for $\theta_t > \underline{\theta}$, $V(z_t, x_t, \theta_t)$ is weakly decreasing in z_t and weakly increasing in x_t.

Recall that $V(z_t, x_t, \theta_t) \geq 0$ for all θ_t. That is, an acquirer can always secure himself a payoff of zero by waiting indefinitely. On the other hand, $R(z_t, x_t, \theta_t) < 0$ for $\theta_t < \underline{\theta}$ which implies that in this range of the economic fundamental it is optimal to wait, i.e. $V(z_t, x_t, \theta_t) = \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t]$. Now let $\theta_t \geq \underline{\theta}$ and consider an increase in θ_t. Both the value of raiding and that of waiting will increase. A simple argument shows that for sufficiently high θ_t, the value of raiding overtakes that of waiting. Assume that for all θ_t

$$\delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t] \geq R(z_t, x_t, \theta_t) > 0$$

Specifically, this implies

$$\sup_{\theta_t} V(z_t, x_t, \theta_t) = \sup_{\theta_t} \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) \,|\theta_t]$$

which contradicts $\delta \in]0,1[$. Assumption A4, i.e. the assumption that

$$R(z_t, x_t, \theta_t) - \delta E[R(z_{t+1}, x_{t+1}, \theta_{t+1})|\theta_t]$$

is strictly increasing in θ_t, ensures that there is a unique crossing since it implies that the value of raiding increases at a higher rate than the value of waiting.[21] In conclusion, for each sequence z^t there exists a unique finite $\widetilde{\theta}(z^t) \in]\underline{\theta}, \infty[$ such that

$$R(z_t, x_t, \widetilde{\theta}(z^t)) = \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1})|\widetilde{\theta}(z^t)]$$

Since $V(z_t, x_t, \theta_t)$ is weakly increasing in x_t, so is $\widetilde{\theta}(z^t)$. Similarly, since $V(z_t, x_t, \theta_t)$ is weakly decreasing in z_t, $\widetilde{\theta}(z^t)$ is weakly decreasing in z_t. This also holds for any future measure of raiders, and thus $\widetilde{\theta}(z^t)$ is also weakly decreasing in z^t. The first-best trigger $\overline{\theta}(z^t)$ is just the value of $\widetilde{\theta}(z^t)$ for the sequence z^t with $z_s = 0$ for all $s \geq t$. Last, note that for all $\theta_t > \underline{\theta}$,and $z_t \geq x_t$, $R(z_t, x_t, \theta_t) > 0$ while $V(z_{t+1}, x_{t+1}, \theta_{t+1}) = 0$.Thus there exists a unique $z_t^* \in [x_t, y_t]$ such that

$$R(z_t^*, x_t, \theta_t) = \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1})\,|\theta_t]$$

It follows from the discussion above that z_t^* is weakly decreasing in θ_t and weakly increasing in x_t ■

B. Proof of Proposition 5

Proof: The first part of the proposition follows immediately from the definitions and Lemma 3. To see the second part, fix a sequence of future cutoff strategies $\{k_s\}_{s=t+1}^{\infty}$ (and thus z^{t+1}) and consider period t. Let a measure μ^i, $i = a, b$ use strategies with cutoff k_t^i with $k_t^a < k_t^b$ and $\mu^a + \mu^b = y_t$. Recall that a cutoff k_t is only an equilibrium strategy if it is optimal for *any* realization of the economic fundamental θ_t. For $\theta_t \in [\underline{\theta}, k_t^a]$ all acquirers wait and the asymmetric strategies can coexist in equilibrium. Similarly, for $\theta_t \in [k_t^b, \overline{\theta}(z^{t+1})]$ all acquirers raid, which is also an equilibrium. Now consider the case where $\theta_t \in [k_t^a, k_t^b]$. Given the considered strategies, a realization in this range prompts a measure μ^a to raid and a measure μ^b to wait. If $\mu^a \geq z_t^*$, the cutoff k_t^b cannot be an equilibrium strategy. Similarly, if $\mu^b < z_t^*$, the cutoff k_t^b cannot be an equilibrium strategy. Thus equilibria in asymmetric strategies do not exist ■

C. Proof of Proposition 8

Before continuing with the analysis, I will state a straightforward lemma that determines the posterior distribution and density of θ given a signal s, that will be useful in the sequel.

[21] To see this, define the mapping $L\Lambda(z_t, x_t, \theta_t) = V(z_t, x_t, \theta_t) - R(z_t, x_t, \theta_t)$. Straightforward manipulation yields $L\Lambda(z_t, x_t, \theta_t) = \max\{0, -R(z_t, x_t, \theta_t) + \delta E[R(z_{t+1}, x_{t+1}, \theta_{t+1})|\theta_t] + \delta E[\Lambda(z_{t+1}, x_{t+1}, \theta_{t+1})|\theta_t]\}$. Under Assumption A3-A4, this defines a mapping L from $\Omega[Y_t \times X_t \times \mathbb{R}]$ into itself which is strictly increasing in θ_t. Furthermore, the mapping L is a contraction with a unique fixed point. Last, note that since $W(z_t, x_t, \theta_t) = \delta V(z_t, x_t, \theta_t)$, it follows that $\Lambda(z_t, x_t, \theta_t) = \max\{0, \Delta(z_t, x_t, \theta_t)\}$.

Lemma 10. *(Posteriors) The posterior density $f_{\theta|s}$ and distribution $F_{\theta|s}$ of θ given some signal s are given by*

$$f_{\theta|s}(\vartheta|s) = \frac{g(\vartheta) f\left(\frac{s-\vartheta}{\sigma}\right)}{\int_{-\infty}^{\infty} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta} \tag{1}$$

$$F_{\theta|s}(\vartheta|s) = \frac{\int_{-\infty}^{\vartheta} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta} = \frac{\int_{\frac{s-\vartheta}{\sigma}}^{\infty} g(s-\sigma u) f(u)\, du}{\int_{-\infty}^{\infty} g(s-\sigma u) f(u)\, du} \tag{2}$$

Proof: Given a signal s, the density of the posterior about θ is given by Bayes' rule as

$$f_{\theta|s}(\theta|s) = \frac{g(\theta) f_{s|\theta}(s|\theta)}{f_s(s)} \tag{3}$$

where f_s and $f_{s|\theta}$ are the densities of s and $s|\theta$ respectively. Now note that, since s is the sum of θ and $\sigma\varepsilon$, its density will be given by the convolution of the densities of their densities, i.e. g and $f_{\sigma\varepsilon}$. Also, noting that $F_{\sigma\varepsilon}(\omega) = F(\omega/\sigma)$ (thus $f_{\sigma\varepsilon}(\omega) = \sigma^{-1} f(\omega/\sigma)$), the unconditional density of s is given by

$$f_s(s) = \sigma^{-1} \int_{-\infty}^{\infty} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta \tag{4}$$

Moreover, to find $f_{s|\theta}$ note that

$$F_{s|\theta}(\varsigma|\theta) = \Pr(s \le \varsigma|\theta) = \Pr\left(\varepsilon \le \frac{\varsigma-\theta}{\sigma}\middle|\,\theta\right) = F\left(\frac{\varsigma-\theta}{\sigma}\right) \Rightarrow \tag{5}$$

$$f_{s|\theta}(\varsigma|\theta) = \frac{d}{ds} F_{s|\theta}(\varsigma|\theta) = \sigma^{-1} f\left(\frac{\varsigma-\theta}{\sigma}\right) \tag{6}$$

because θ and ε are independent. Thus, substituting (4) and (6) in (3), the density of θ given s is:

$$f_{\theta|s}(\vartheta|s) = \frac{g(\vartheta) f\left(\frac{s-\vartheta}{\sigma}\right)}{\int_{-\infty}^{\infty} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta}$$

Therefore the posterior distribution is

$$\begin{aligned}
F_{\theta|s}(\vartheta|s) &= \int_{-\infty}^{\vartheta} f_{\theta|s}(\theta|s) d\theta \\
&= \int_{-\infty}^{\vartheta} \frac{g(\theta) f\left(\frac{s-\theta}{\sigma}\right)}{\int_{-\infty}^{\infty} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta} d\theta \\
&= \frac{\int_{-\infty}^{\vartheta} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} g(\theta) f\left(\frac{s-\theta}{\sigma}\right) d\theta}
\end{aligned}$$

The second part of equality (2) follows by performing the transformation $u = \sigma^{-1}(s-\theta)$ ■

To formally state the proposition of the existence of a unique perfect Bayesian equilibrium in monotone strategies, assume for now that the waiting value is well defined as a function of current information and strategies. That this is indeed the case will be verified shortly.

Consider a simplified associated game $\Gamma_t^*(T)$, where it is assumed that the received signal is a sufficient statistic of the state and θ is drawn from a uniform distribution on the real line. Throughout, an asterisk will denote quantities pertaining to the associated game. Although the prior of θ is an improper distribution (has infinite probability mass), it is possible to apply Lemma 10 by normalizing the prior density to one, i.e. $g(\theta) = 1$. The density of the posterior is then given by $f^*_{\theta|s}(\theta|s) = \sigma^{-1} f\left(\frac{s-\theta}{\sigma}\right)$ and the distribution by $F^*_{\theta|s}(\theta|s) = 1 - F\left(\frac{s-\theta}{\sigma}\right)$.

Let $\Delta^*_\sigma(s,k)$ denote the expected payoff gain to "waiting" with respect to the posterior after having received signal s, and believing that all other players use strategies with cutoffs k.[22] This is given by

$$\begin{aligned}\Delta^*_\sigma(s,k) &\equiv E^*_{\theta|s}\left[\Delta\left(y\left[1-F\left(\frac{k-\theta}{\sigma}\right)\right],x,s\right)\right]\\ &= \int_{-\infty}^{\infty}\Delta\left(y\left[1-F\left(\frac{k-\theta}{\sigma}\right)\right],x,s\right)\sigma^{-1}f\left(\frac{s-\theta}{\sigma}\right)d\theta \qquad (7)\end{aligned}$$

For comparison, consider the underlying game at time t, and denote by $\Delta_\sigma(s_t,k_t)$ the expected payoff gain to waiting when signal s_t has been observed and all other acquirers use cutoffs k_t. By Lemma 10, this is given by

$$\begin{aligned}\Delta_\sigma(s_t,k_t) &\equiv E_{\theta|s}\left[\Delta\left(y_t\left[1-F\left(\frac{k_t-\theta_t}{\sigma}\right)\right],x_t,\theta_t\right)\right]\\ &= \frac{\int_{-\infty}^{\infty}\Delta\left(y_t\left(1-F\left(\frac{k_t-\theta}{\sigma}\right),x_t,\theta\right)\right)g(\theta)f\left(\frac{s_t-\theta}{\sigma}\right)d\theta}{\int_{-\infty}^{\infty}g(\theta)f\left(\frac{s_t-\theta}{\sigma}\right)d\theta} \qquad (8)\end{aligned}$$

The differences between (7) and (8) are twofold. First, in (7), the signal s replaces the economic fundamental θ. Second, the posterior distributions over the economic fundamental are generated by different prior beliefs. All other properties are shared.

Assume for now that the functions $\Delta_\sigma(s_t,k_t)$ and $\Delta^*_\sigma(s_t,k_t)$ are well defined, and that all players receiving identical signals would have identical beliefs about the exact shapes of the functions. With these definitions in place, the following lemmata needed for the proof of the uniqueness result can be stated:

Lemma 11. *(Uniqueness in Associated Game) For any history $h_t \in H_t$, there exists a unique cutoff signal s^*_t in the associated static game Γ^*_t such that: $\Delta^*_\sigma(s^*_t,s^*_t) = 0$, $\Delta^*_\sigma(s_t,s^*_t) > 0$ for $s_t < s^*_t$ and $\Delta^*_\sigma(s_t,s^*_t) < 0$ for $s_t > s^*_t$.*

Proof: To prove Lemma 11, two separate results need to be established. First, it is shown that there is a unique signal such that indifference obtains exactly when receiving signal $s_t = s^*_t$. Second, it is shown that for lower signals waiting is optimal, while for higher signals raiding is optimal.

First rewrite $\Delta^*_\sigma(s,k)$, by changing variables using $z = y\left[1-F\left(\frac{s-\theta}{\sigma}\right)\right]$:

$$\Delta^*_\sigma(s,k) = \int_0^y \Delta\left(z,x,k\right)y^{-1}dz$$

[22] Since the associated game is essentially static, time subscripts are omitted for ease of notation. This should cause no confusion.

For $k = s$,

$$\Delta^*_\sigma(s,s) = \int_0^y \Delta\left(z,x,s\right) y^{-1} dz$$

In other words, the function $\Delta^*_\sigma(s,k)$ has been rewritten such that it is an integral over a uniform distribution of z over $[0,y]$. But generically, there is a unique s^* that solves

$$\int_0^y \Delta\left(z,x,s^*\right) y^{-1} dz = 0$$

Thus that there is exactly one cutoff signal s^* at which an agent is exactly indifferent between raiding and waiting. It now has to be verified that there exists an equilibrium where the agent raids whenever $s > s^*$ and waits whenever $s < s^*$. In order to do this, recall that the game displays *action single crossing* (follows from Lemma 3), *state monotonicity* (follows from Lemma 2) and that the noise distribution has the monotone likelihood ratio property (Assumption A7).

The expected payoff gain to waiting, given signal s, when all other players use cutoffs k is given by

$$\begin{aligned}\Delta^*_\sigma(s,k) &\equiv \int_{-\infty}^{\infty} \Delta\left(y\left(1-F\left(\frac{k-\theta}{\sigma}\right)\right),x,s\right)\sigma^{-1} f\left(\frac{s-\theta}{\sigma}\right) d\theta \\ &= \int_{-\infty}^{\infty} \Delta\left(y\left[1-F(-m)\right],x,s\right) f\left(\frac{s-k}{\sigma}-m\right) dm\end{aligned}$$

by changing variables so that $m = \sigma^{-1}(\theta - k)$. Now rewrite the above expression as

$$\begin{aligned}\Delta^*_\sigma(s,k) &= \widetilde{\Delta}(s,k,s') \\ &\equiv \int_{-\infty}^{\infty} \gamma(m,s')\varphi(s,m) dm\end{aligned}$$

where

$$\begin{aligned}\gamma(m,s') &= \Delta(1-F(-m),x,s') \\ \varphi(s,m) &= f\left(\frac{s-k}{\sigma}-m\right)\end{aligned}$$

Because of the monotone likelihood ratio property, $\widetilde{\Delta}(.,k,s')$ preserves the single crossing property of $\Delta(z,x,\theta)$ by a result by Athey (2002). That is, there exists $s^*(k,s')$ such that

$$\begin{aligned}\widetilde{\Delta}(s,k,s') < 0 \quad if \quad s > s^*(k,s') \\ \widetilde{\Delta}(s,k,s') > 0 \quad if \quad s < s^*(k,s')\end{aligned}$$

By state monotonicity, $\widetilde{\Delta}(s,k,s')$ is strictly decreasing in s'. Now let $s > s'$ and suppose that

$$\widetilde{\Delta}(s,k,s) = 0$$

It follows that

$$\widetilde{\Delta}(s',k,s') > \widetilde{\Delta}(s',k,s) > \widetilde{\Delta}(s,k,s) = 0$$

where the first inequality comes from state monotonicity and the second comes from the action single crossing property. A symmetric argument holds for $s < s'$. This implies that there exists a best response function $\beta : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{aligned} \Delta^*_\sigma(s,k) < 0 \quad if \quad s > \beta(k) \\ \Delta^*_\sigma(s,k) = 0 \quad if \quad s = \beta(k) \\ \Delta^*_\sigma(s,k) > 0 \quad if \quad s < \beta(k) \end{aligned}$$

But there exists a unique s^* that solves

$$\Delta^*_\sigma(s^*,s^*) = \int_0^y \Delta(z,x,s^*)y^{-1}dz = 0$$

Therefore $\beta(k) = k$. It has thus been shown that with a uniform prior, there exists a unique equilibrium in cutoff strategies such that

$$a_k(s) = \begin{cases} 1 & if \quad s > s^* \\ 0 & if \quad s < s^* \end{cases}$$

This proves Lemma 11 ■

Lemma 12. *(Limit Uniqueness in Underlying Game) For any history $h_t \in H_t$, as $\sigma \to 0$, $\Delta_\sigma(s_t, s_t - \sigma\xi) \to \Delta^*_\sigma(s_t, s_t - \sigma\xi)$ uniformly.*

Proof: It was shown in Lemma 11 that in the associated game with uniform prior and private values, there is a unique equilibrium sequence of cutoffs. What remains to be shown is that the game with general prior and private values comes arbitrarily close to the associated private values uniform priors game, as noise vanishes.

Recall that $\Delta_\sigma(s,k)$ is

$$\begin{aligned} \Delta_\sigma(s,k) &= E_{\theta|s}\left[\Delta\left(y\left[1-F\left(\frac{k_t-\theta_t}{\sigma}\right)\right], x_t, \theta_t\right)\right] \\ &= \int_{-\infty}^{\infty} \Delta\left(y\left[1-F\left(\frac{k_t-\theta}{\sigma}\right)\right], x_t, \theta\right) dF_{\theta|s}(\theta|s) \end{aligned} \tag{9}$$

where $F_{\theta|s}(\theta|s)$ is the posterior distribution. To do a change of variables using $z = y\left[1-F\left(\frac{s-\theta}{\sigma}\right)\right]$, first note that

$$\theta = k - \sigma F^{-1}\left(\frac{y-z}{y}\right)$$

Next, let $\Psi_\sigma(z;s,k)$ be the posterior distribution evaluated at this θ. From Lemma 10 it follows that

$$\Psi_\sigma(z;s,k) = F_{\theta|s}\left(k - \sigma F^{-1}\left(\frac{y-z}{y}\right)|s\right) = \frac{\int_{\frac{s-k}{\sigma}+F^{-1}\left(\frac{y-z}{y}\right)}^{\infty} g(s-\sigma u)f(u)du}{\int_{-\infty}^{\infty} g(s-\sigma u)f(u)du}$$

Therefore (9) becomes,

$$\Delta_\sigma(s,k) = \int_0^y \Delta\left(z, x_t, k - \sigma F^{-1}\left(\frac{y-z}{y}\right)\right) d\Psi_\sigma(z;s,k)$$

Recall from the proof of Lemma 11 that for the associated game

$$\Delta_\sigma^*(s,k) = \int_0^y \Delta\left(z,x,k\right) y^{-1} dz = \int_0^y \Delta\left(z,x,s\right) d\Psi_\sigma^*(z;s,k)$$

where $\Psi_\sigma^*(z;s,k) = F_{\theta|s}^*\left(k - \sigma F^{-1}\left(\frac{y-z}{y}\right)|s\right) = 1 - F\left(\frac{s-k}{\sigma} + F^{-1}\left(\frac{y-z}{y}\right)\right)$. Thus $\Psi_\sigma^*(z;s,s) = \frac{y-z}{y}$, which is the distribution function of the uniform distribution on $[0,y]$.

Returning to $\Psi_\sigma(z;s,k)$, note that for some small $|\xi|$

$$\Psi_\sigma(z;s,s-\sigma\xi) = \frac{\int_{\xi+F^{-1}\left(\frac{y-z}{y}\right)}^\infty g(s-\sigma u)f(u)du}{\int_{-\infty}^\infty g(s-\sigma u)f(u)du} \underset{\sigma\to 0}{\to} 1 - F\left(\xi + F^{-1}\left(\frac{y-z}{y}\right)\right) = \Psi_\sigma^*(z;s,s-\sigma\xi)$$

Therefore, $\Delta_\sigma(s,s-\sigma\xi) \to \Delta_\sigma^*(s,s-\sigma\xi)$ continuously as $\sigma \to 0$.

What remains to be shown is that $\Delta_\sigma(s,s-\sigma\xi) \to \Delta_\sigma^*(s,s-\sigma\xi)$ uniformly as $\sigma \to 0$. In other words, one must ensure that the equivalence of the two games is not a result of a discontinuity at $\sigma = 0$. Instead of showing uniform convergence directly, I will proceed by showing convergence with respect to the uniform convergence norm. Convergence in this norm implies uniform convergence. First, note that there exist extreme signals $\underline{s}$ and $\overline{s}$ such that for all k: $\Delta_\sigma(s,k) > 0$ for $s < \underline{s}$ and $\Delta_\sigma(s,k) < 0$ for $s > \overline{s}$. This follows from the existence of dominance regions $[-\infty,\underline{\theta}]$ and $[\overline{\theta},\infty]$, where there is a unique optimal action. One can thus pick any pair $\underline{s}$ and $\overline{s}$ such that $\underline{s} < \underline{\theta}$ and $\overline{s} > \overline{\theta}$, and restrict attention to the compact interval $S \equiv [\underline{s},\overline{s}]$. Since S is compact and the second argument of the Δ_σ function is continuous with respect to s (i.e. the function $s - \sigma\xi$), the set $K \equiv [\underline{s} - \sigma\xi, \overline{s} - \sigma\xi]$ is also compact. Hence $\Delta_\sigma(s,k)$ takes values in a compact set. Next, define the sup-norm (or *uniform convergence norm)*

$$||\Delta|| \equiv \sup_{s,k}\{|\Delta(s,k)|\}$$

It has to be shown that $\Delta_\sigma(s,k)$ is continuous in the uniform convergence topology. I start by showing continuity of $\Delta_\sigma(s,k)$ with respect to the Euclidean metric. Fix s',k'. Since the function is continuous in both arguments, it follows that

$$\begin{aligned}
&\forall\varepsilon_1 > 0, \exists\delta_1 : |s-s'| < \delta_1 \Rightarrow |\Delta_\sigma(s,k) - \Delta_\sigma(s',k)| < \varepsilon_1 \forall k \\
&\forall\varepsilon_2 > 0, \exists\delta_2 : |k-k'| < \delta_2 \Rightarrow |\Delta_\sigma(s,k) - \Delta_\sigma(s,k')| < \varepsilon_2 \forall s
\end{aligned}$$

This in turn implies that

$$\sqrt{(s-s')^2 + (k-k')^2} < \overline{\delta} \equiv \sqrt{\delta_1^2 + \delta_2^2}$$

But then by the triangle inequality it follows that

$$\begin{aligned}
|\Delta_\sigma(s,k) - \Delta_\sigma(s',k')| &= |\Delta_\sigma(s,k) - \Delta_\sigma(s',k) + \Delta_\sigma(s',k) - \Delta_\sigma(s',k')| \\
&\le |\Delta_\sigma(s,k) - \Delta_\sigma(s',k)| + |\Delta_\sigma(s',k) - \Delta_\sigma(s',k')| \\
&\le \varepsilon_1 + \varepsilon_2 \equiv \varepsilon
\end{aligned}$$

and continuity with respect to the Euclidean metric follows. Denoting by $\mathbf{C}(S \times K)$ the space of continuous functions on $S \times K$, it follows that $\Delta_\sigma(s,k) \in \mathbf{C}(S \times K)$. But showing uniform convergence is equivalent to showing that as $\sigma \to 0$,

$$||\Delta_\sigma - \Delta_\sigma^*|| = \sup_{s,k} \{|\Delta_\sigma(s,k) - \Delta_\sigma^*(s,k)|\} \to 0$$

with respect to the sup-norm. By substituting for the relevant functions and taking limits, the result follows ■

Lemma 11 states that any static associated game has a unique Bayesian equilibrium. Lemma 12 shows that when private information is very precise, any finite horizon version of the underlying game becomes arbitrarily close to a sequence of simplified static associated games. In other words, as $\sigma \to 0$, the period t expected relative payoff function in the underlying game, converges uniformly to the relative payoff function of some simplified static game which has a unique Bayesian equilibrium in monotone strategies. Having established these results, the sought proof follows:

Proof of Proposition 8: It is first established that $\Delta_\sigma(s_t, k_t)$ and $\Delta_\sigma^*(s_t, k_t)$ are well defined. Consider the truncated game, where play is exogenously terminated after some period T. At time T, optimality dictates that remaining acquirers raid for all signals that convince them of receiving a non-negative payoff. Note that this is irrespective of what other players do (i.e. independent of z_T). Now consider the (possibly trivial) decision at time $T-1$. Because equilibrium actions are well defined (and unique) at time T, the expected waiting value at time $T-1$ is well defined. But then, so is the expected net waiting value $\Delta_\sigma(s_{T-1}, k_{T-1})$. The problem to be solved at time $T-1$ is essentially a static game as the one considered in the Lemma 11, and thus there exists a unique equilibrium with cutoff s_{T-1}^*. Having established uniqueness at time $T-1$, assume that at time $\tau < T-1$ there exists a unique sequence $\{s_t^*\}_{t=\tau+1}^T$ of equilibrium cutoffs. With this inductive assumption, the next step is to show that at time $\tau - 1$ there exists a unique sequence of equilibrium cutoffs $\{s_t^*\}_{t=\tau}^\infty$. To see this, recall that the expected payoff gain from waiting is given by $\Delta_\sigma(s_\tau, k_\tau)$. This function shares all the properties of the function $\Delta_\sigma(s_{T-1}, k_{T-1})$ and thus there exists a unique equilibrium in monotone strategies with cutoff s_τ^*. Having shown uniqueness for arbitrary finite horizon version of the model, the infinite horizon game is considered. First note that the optimal strategy at any point in time optimally trades off the value of waiting with the value of raiding, i.e. the function $\Delta_\sigma(s_t, k_t)$. Clearly, $\Delta_\sigma(s_t, k_t)$ converges to a unique limit as $T \to \infty$, since both the value of raiding and that of waiting are bounded monotone functions of T. But then $s_t^*(T) \to s_t^*(\infty)$ as $T \to \infty$, where $s_t^*(T)$ is the equilibrium cutoff in period t in the game truncated after period T and $s_t^*(\infty)$ is the equilibrium cutoff at time t in the infinite horizon game ■

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