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TIME OF THE ESSENCE

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TIME OF THE ESSENCE

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ABSTRACT. In most industries, ranging from information systems development to construction, an overwhelming proportion of projects are delayed beyond estimated completion time. This fact constitutes somewhat of a puzzle for existing theory. The present paper studies project delays and optimal contracts under moral hazard in a setting with time to build. Within this setup, project delays are found to be most likely to happen at early stages of development, and intimately connected to the degree of commitment of the procurer and the class of contracts that can be enforced. The first-best, optimal spot contracting and optimal long-term contract scenarios are analyzed, as well as commonly encountered additional constraints on the long-term contract.

"Most attempts to explain why deadlines are missed [...] go no farther than Murphy's often-quoted aphorism" - Musgrove (1985).

1. INTRODUCTION

Explicit stipulation of delivery times are practically universal in contracts. Examples include labor contracts within firms, sub-contracting of parts of a larger project to other firms, and procurement contracts for large-scale projects such as weapons systems and infrastructure. In some of these cases, a deadline is determined exogenously. For example, if a production process involves the use of a perishable input, failing to meet a production deadline might mean the loss of the input in question. Another example is a situation in which one of the parties is bound by a contract with a third party. But in many situations, a deadline is imposed endogenously by one of the parties.

In this paper, I propose a framework, within which delivery time and its connection to asymmetric information and incentive considerations can fruitfully be analyzed. Specifically, I provide a simple setting in which the agent benefits from procrastinating and thus delaying delivery beyond the efficient point in time, unless given incentives to do otherwise. In this setting, I analyze optimal contracts under different sets of assumptions on information and commitment.

In terms of policy, the present analysis if firmly normative. It suggests ways in which the significant problem of time overruns can be ameliorated, arguing that the failure of real-world

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contracts in effectively controlling production schedules may be the undesirable outcome of not effectively incorporating incentive considerations into contract design.

The specific setup is as follows. A risk neutral principal delegates the completion of a long-term project to a risk averse agent. Project completion involves completing a number of separate phases or tasks in a fixed, prespecified order. The benefits from the project accrue on successful completion of the last task and are discounted. Progress through the stages of the project is random and partially controlled by the agent, who exerts effort influencing whether or not a task is successfully completed, or must be attempted again.

The analysis breaks down in four different parts. First, I analyze the first-best benchmark in which the principal can disregard incentive considerations. In the first-best contract, the agent is fully insured, but kept to his reservation utility. Furthermore, since the value of the project is increasing in progress (i.e. in the number of completed tasks), the contract induces an increasing sequence of effort. This means that failures (and thus project delays) are more likely to occur at early stages of development. Last, since effort is increasing in progress and the agent's participation constraint is binding, wages are also increasing in progress.

Second, I consider a setting in which the agent's effort is unobservable and where the principal cannot commit to any future contract (or to any continuation of a contract currently on offer), i.e. the relationship is governed by a sequence of spot contracts. In this setup, the agent is no longer fully insured and is therefore exposed to some risk. While it follows from revealed preferences that effort is lower than the first-best level at all stages of the project, it is still the case that effort is increasing in progress. Last, as in the first-best contract, the optimal continuation contract is memoryless in the sense that past history does not influence the contract on offer at any given time for the completion of a given task.

Third, and moving to the opposite extreme, I consider a setting in which the principal can credibly commit to any long-term contract. In this setting, it is shown that the optimal contract has memory and that the agent is rewarded (or punished) not only in a given period based on that period's outcome, but that the continuation contract is designed to reinforce correct incentives. In effect, the ability to write long-term contracts facilitates intertemporal smoothing of the agent's wages, thereby relaxing the incentive problem and making it cheaper for the principal to induce the agent to exert effort.

Finally, I consider the effects of a number of different constraints on the optimal long-term contract which are often observed in practice. Specifically, I consider the effects of limited liability constraints, limits imposed by budgetary horizons, the effects of non-commitment of the agent and last, the effects of non-enforcement of penalty clauses and their replacement by liquidated damages clauses. I argue that under all these constraints, the principal's ability to provide incentives is hampered, although for different reasons. Limited liability and non-enforcement of penalty clauses both restrict the agent's utility to lie within certain bounds. In the former case, utility cannot fall below a certain threshold while in the latter, the restriction is on the variability of the agent's utility. Non-commitment of the agent implies that the single intertemporal participation constraint is replaced by a sequence of constraints, one for each possible history. Last, budgetary horizons limit the principal's ability to smoothe the agent's utility over time, effectively exposing him to a suboptimal amount of risk.

Stylized Facts. In the empirical literature, as well as in the popular press, the notion of delays and time overruns has received considerable attention. In a study of information

systems development, Jenkins, Naumann and Wetherbe (1984) found that 90% of projects were delayed. Similarly, Van Genuchten (1991) found that in software development, 50% of milestones were not met. Research at the Rand corporation by Marshall and Meckling (1962) showed that the ratio of actual completion time to estimated completion time for a sample of large-scale weapons systems was 1.5, while Peck and Scherer (1962) found a ratio of 1.36 for another sample of weapons development programs. Mansfield, Schnee and Wagner (1971) report similar ratios for drugs development, ranging from 1.61 to 2.95.

Casual observation suggests that the occurrence of project delays are pervasive in all areas of development where time plays an important role. This is true both for industries subject to substantial uncertainty, such as information systems, pharmaceuticals and biotech, and for lower uncertainty industries, such as construction. Despite these observations, economic theory has been curiously silent about possible explanations. At first glance, one may be tempted to reduce the problem of project delays to irrational decision making or overly optimistic forecasts. But once it is recognized that delays occur consistently, one is faced with the question of why this information is not properly taken into account when performing estimates of project duration.

Thus, estimates of completion time are consistently biased. It is worth fleshing out what this observation implies. Namely, that given any amount of information on the project, and any amount of experience of similar projects previously executed, the planner is likely to get the estimated completion time wrong. I shall argue that one reason for this bias is the total absence of incentive considerations in currently used forecasting techniques. A commonly used technique is the so-called Critical Path Method (CPM).[1] The CPM is an accounting tool that helps schedule the development process. It clarifies the precedence relations among tasks by showing which tasks depend on other tasks having been completed, which tasks can or should be executed in sequence or in parallel, etc. But ultimately, a planner using the CPM still has to perform an estimate of how much time the completion of each task requires, and in performing these estimates, incentives play no role.

Related Literature. Although the design of contracts that involve deadlines for completion of projects is of significant practical interest, the theoretical literature on the subject is surprisingly small, as evidenced by the opening quote. Only a few papers have dealt explicitly with contracting for delivery in long-term projects. Cukierman and Shiffer (1976) consider the effects of commonly used payment schedules on an agent's incentive to deliver at the efficient point in time. In particular, they show that if the payment is exogenously determined, and hence not contingent on the date of delivery, the agent will benefit from delaying. However, their model features no uncertainty or asymmetry of information. Toxvaerd (2003) considers a continuous time adverse selection model with time to build, in which the agent's production function is parametrized by a privately known efficiency parameter. In this framework, it is shown that the optimal contract effectively uses delivery time to screen agents, and that all but the most efficient agent delivers inefficiently late. The optimal contract therefore induces inefficient delay in order to reduce informational rents.

The problem of delays and its connection to asymmetric information has been treated in another context by O'Donoghue and Rabin (1999). They explain delays as the effect of the

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[1] A related method is the Program Evaluation and Review Technique (PERT).

agent's propensity to procrastinate. In contrast, the model developed here assumes that the agent's preferences are fully time consistent.

Finally, another related and highly humorous contribution is that of Musgrove (1985). Based on casual observation, he presents a model in which a workload can be subdivided into smaller projects at will. He shows within this framework why things should take 2.71828 times as long as expected.

The remainder of the paper is organized as follows. Section 2 sets out the basic model. In section 3, the first-best benchmark is analyzed and the optimal contract characterized. Furthermore, some basic comparative statics results of the model are presented. In section 4, the spot contracting scenario in which the principal has no commitment at all is analyzed. Section 5 analyzes the optimal long-term contract and partially characterizes it. Section 6 contains a discussion of different constraints on the optimal long-term contract and section 7 contains concluding comments.

2.   The Model

A principal delegates the completion of a project to an agent. The project involves N distinct phases or tasks which have to be performed in a fixed, prespecified order. Work on task i cannot commence before task $(i-1)$ has been completed. Attempting to complete a task takes one period. For each task, the agent chooses the probability $q \in [0,1]$ of successfully completing it. Throughout, the terms *effort* and *probability of success* will be used interchangeably. If task i fails, it has to be attempted again until success occurs. Then the agent can attempt to complete task $(i+1)$. On completion of task N, the principal receives value $R > 0$. Both the principal and the agent discount the future with factor $\delta \in [0,1[$. Denoting by T_i the completion date of task i, the principal's utility gross of wage payments is given by

$$\delta^{T_N} R$$

The agent's utility in a given period is given by the separable function

$$u(w) - \psi(q)$$

where w is a wage transfer from the principal. The disutility of effort is an increasing convex function, with $\psi' > 0$, $\psi'' > 0$ and $\psi''' \geq 0$, while utility of wealth is an increasing concave function, so $u' > 0$ and $u'' > 0$.[2] Last, it is assumed that

$$\lim_{q \to 1} \psi(q) = \infty$$

The date of completion of any task (and thus, also the completion of the project) is a random variable. The state of the project, i.e. the completed number of tasks, is the basic state variable of the model and is a simple counting process. Ceteris paribus, the principal wishes the agent to finish the project as soon as possible, while the agent, having convex disutility of effort, has an incentive to stretch completion time.

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[2] A similar model is considered in Grossman and Shapiro (1986), but in continuous time and solved as a pure decision theoretic model. Also, similar frameworks have been considered by Sobel (1992) and Kremer (1993) who both assume that if task i fails, all previously completed tasks $j \leq i - 1$ must be repeated. Importantly, they both abstract from incentive considerations.
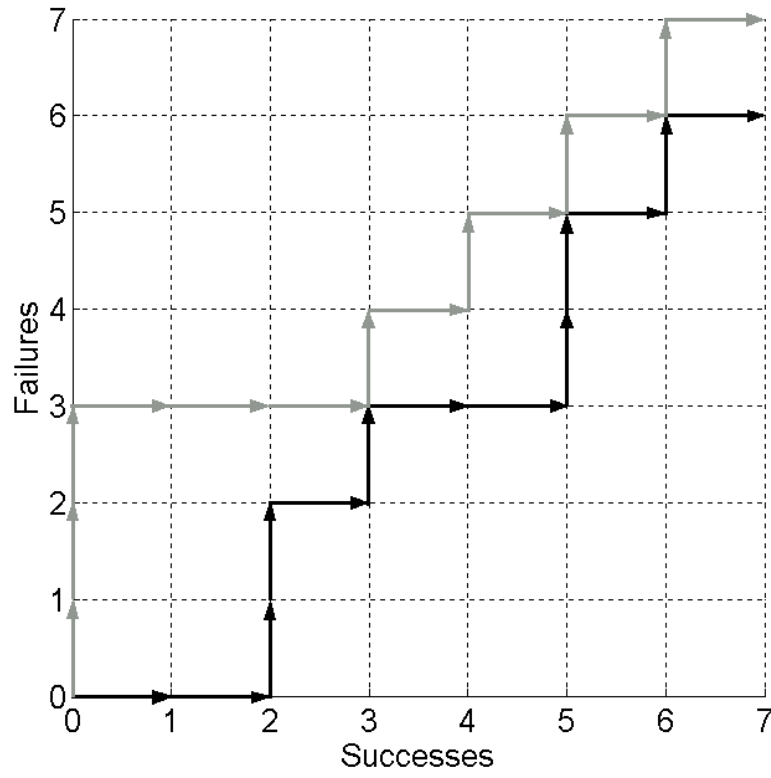
Figure 1: Progress and delay, $N = 7$. Each vertex corresponds to one period. Movement in the horizontal direction denotes a success; movement in the vertical direction denotes a failure.

The number of tasks N to be completed will be referred to as the *scale* of the project, and the steepness of ψ will be referred to as the *difficulty* of the project.

Figure 1 shows a *progress grid*, and two possible paths for the case $N = 7$. Movement in the horizontal direction denotes success while movement in the vertical direction denotes failure. Each vertex corresponds to one period of time. Hence, the path shown in black achieves project completion at time $T_7 = 13$ while the path in grey achieves completion at time $T_7 = 14$. As is clear from the figure, this does not imply that the black path displays smaller delay on each task.

Before commencing the formal analysis of the model, it should be noted that it is assumed throughout that the principal can perfectly control the agent's access to the credit market. In particular, it is assumed that the agent must consume a given period's wage in its entirety in that same period, and that the agent has no outside source of funds.

3. FIRST-BEST BENCHMARK

First, consider the principal's problem in the absence of asymmetric information. Denoting by V_i the value to the principal of having completed $(i - 1)$ tasks, he solves the following problem for all $i = 1, ..., N$:[3]

$$\max_{\{q_i, \overline{w}_i, \underline{w}_i\}} \left\{ -q_i \overline{w}_i - (1 - q_i)\underline{w}_i + \delta \left[q_i V_{i+1} + (1 - q_i)V_i \right] \right\} \tag{1}$$

subject to

$$q_i u(\overline{w}_i) + (1 - q_i)u(\underline{w}_i) - \psi(q_i) \geq 0 \tag{2}$$

where $V_{N+1} \equiv R$. Throughout, an upper bar denotes transfers for successes and lower bar denotes transfers for failures.

Condition (2) is simply the agent's individual rationality constraint, which ensures that he gets at least his reservation utility (which is normalized to zero).

Denoting by λ_i the multiplier on (2), the optimal wages are characterized by the following first-order conditions with respect to wages:

$$\lambda_i u'(\overline{w}_i) - 1 = 0 \tag{3}$$

$$\lambda_i u'(\underline{w}_i) - 1 = 0 \tag{4}$$

In turn, optimal effort is given by the first-order condition with respect to success probability:

$$\delta \left[V_{i+1} - V_i \right] - \left[\overline{w}_i - \underline{w}_i \right] + \lambda_i \left[u(\overline{w}_i) - u(\underline{w}_i) - \psi'(q_i) \right] = 0 \tag{5}$$

From (3)-(4) and the fact that $u'' > 0$, it follows that $\overline{w}_i = \underline{w}_i = w_i$, which in turn reduces (5) to

$$\delta \left[V_{i+1} - V_i \right] = \lambda_i \psi'(q_i) \tag{6}$$

The interpretation is simply that under the first-best contract, the agent is fully insured and since $\lambda_i > 0$, is kept to his reservation utility. Furthermore, induced effort is such that the principal's marginal benefit of success is equal to the marginal cost of inducing the chosen effort.

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[3]In what follows, the qualifier *for all i* will be omitted.

Next, I characterize how the contract evolves with progress. First, note that since (2) is binding, it follows from (1) that

$$V_i = \max_{q_i} \left\{ -q_i \overline{w}_i - (1 - q_i)\underline{w}_i + \delta \left[q_i V_{i+1} + (1 - q_i)V_i \right] + \lambda_i \left[q_i u(\overline{w}_i) + (1 - q_i)u(\underline{w}_i) - \psi(q_i) \right] \right\}$$

Solving for V_i yields

$$V_i = \max_{q_i} \left\{ \frac{q_i \delta V_{i+1} - q_i \overline{w}_i - (1 - q_i)\underline{w}_i + \lambda_i \left[q_i u(\overline{w}_i) + (1 - q_i)u(\underline{w}_i) - \psi(q_i) \right]}{1 - \delta(1 - q_i)} \right\}$$

and thus optimal effort on task i is given by

$$q_i^* \in \arg\max_{q_i} \left\{ \frac{q_i \delta V_{i+1} - q_i \overline{w}_i - (1 - q_i)\underline{w}_i + \lambda_i \left[q_i u(\overline{w}_i) + (1 - q_i)u(\underline{w}_i) - \psi(q_i) \right]}{1 - \delta(1 - q_i)} \right\}$$

Taking the first-order condition with respect to effort and solving for V_{i+1} yields

$$V_{i+1} = \frac{\overline{w}_i(1 - \delta) - \underline{w}_i - \lambda_i \left[(1 - \delta)u(\overline{w}_i) - u(\underline{w}_i) \right] - \lambda_i \left[\delta\psi(q_i) - \psi'(q_i)\left(1 - \delta(1 - q_i)\right) \right]}{\delta(1 - \delta)}$$

The right-hand side of this expression is increasing in effort q_i.[4] But since the project's value is increasing in progress, i.e. $V_{i+1} > V_i$, this implies that

$$q_{i+1} > q_i$$

for all tasks $i = 1, ..., N$. Thus effort is strictly increasing in progress.

The intuition for this result is straightforward. The more progress there is, the more valuable the (remaining) project becomes. And since delay thus becomes more costly, the higher is the induced effort (i.e. probability of success). An immediate consequence is that failure and delay is more likely to occur at the early stages of the project.

Next, using (6) and substituting for λ_i, it follows that

$$\frac{\psi'(q_i)}{u'(w_i)} = \delta \left[V_{i+1} - V_i \right] > \delta \left[V_i - V_{i-1} \right] = \frac{\psi'(q_{i-1})}{u'(w_{i-1})} \tag{7}$$

where the inequality in (7) follows from $q_i > q_{i-1}$ and the fact that the individual rationality constraint (2) then implies that $w_i > w_{i-1}$. Inequality (7) implies that not only does effort and wages increase in progress, but that they increase at an increasing rate.

Note that the first-best contract is memoryless, in the sense that the contract offered by the principal for the completion of a given task i is independent of the past history of failures and successes on all previous tasks, and the number of failed attempts on that particular task. Thus the contract offered at point (2,0) on figure 1 is identical to that offered at point (2,3). For later comparison, note that in the first-best contract, not only is the agent supplied with full insurance, but there is no need to smoothe his consumption over time. As shall become clear in the analysis in sections 4-6, these two features become important determinants of delivery time once moral hazard and commitment is introduced to the model.

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[4]This follows directly by inspection of its derivative with respect to q_i.

Summing up, the first-best contract implements an increasing sequence of effort (i.e. increasing in progress), fully insures the agent (thus there is optimal risk sharing) and keeps the agent to his reservation utility (and so, wages are increasing in progress). Furthermore, the first-best contract is memoryless, in the sense that past history does not influence the contract offered at any point in time for the completion of a given task.

Expected Completion Time. Next, I will characterize the expected completion time, as well as present some simple comparative statics results. First note that by construction, the principal's problem is stationary, in turn implying that the optimal policy of implemented efforts is stationary. Thus each time task i is attempted, the same effort level q_i will be implemented and $q_{ij} = q_{ik}$ for all $j \neq k$, where q_{ij} is effort on attempt j on task i. But then work on task i constitutes a sequence of independent Bernoulli trials with probability of success q_i, constant over trials. Denote by X_i be the number of trials on task i until the first success. Then the mean and variance of termination time of task i are given by

$$E[X_i] = q_i^{-1}$$
$$V[X_i] = (1 - q_i)q_i^{-2}$$

The project as a whole is thus characterized by a sequence of random variables $\{X_i\}_{i=1}^{N}$ that follow the geometric distribution with parameter q_i, which is increasing in i. The sum of the random variables X_i represents the total number of trials in all stages until the project is successfully completed. Hence the expected number of trials until overall project completion is

$$E[T_N] = E\left[\sum_{i=1}^{N} X_i\right] = \sum_{i=1}^{N} q_i^{-1} \tag{8}$$

Since it is assumed that the project is viable, and thus that $q_i > 0$ for all i, it follows that $E[T_N]$ is strictly increasing and convex in N, and bounded below by N. In other words, the expected time of project completion is an increasing convex function of the scale of the project.

The more sharply $\psi(q)$ increases in effort q, the more expensive it becomes to implement effort. Consequently, as the difficulty of the project increases, the induced effort q will decrease. But expected termination time is a decreasing convex function of effort. Thus an increase in the difficulty of the project induces a more than proportionate increase in the expected completion time. Furthermore, the variance of T_N is a decreasing convex function of effort levels q_i.

Given gross project value R, the net value of the project decreases in N, which in turn decreases the implemented efforts q_i. Thus the larger the scale of the project N, the more variable is completion time. Similarly, an increase in project value R increases the sequence of implemented efforts. Last, since effort q_i is increasing in progress, the estimate of remaining development duration becomes less variable the closer the project is to completion. In fact, the increasing sequence of implemented efforts implies that as progress is made, expected completion time of a given task decreases, in the sense of first-order stochastic dominance.

4. Spot Contracts

Next, I turn to a scenario where there is moral hazard in the agent's effort choice and incentive compatibility thus cannot be ignored. For simplicity, assume that the principal has

no ability to commit whatsoever, and thus offers the agent a new contract in each period. In other words, if the agent fails on task i, the principal offers a new contract for the next attempt on task i, while if there is success, the agent is offered a contract for the first attempt on task $(i+1)$. It is important to note that under spot contracting, the principal's problem is stationary in attempts on a given task. Since by assumption there is no commitment, the contract offered after a failure on a given task is identical to the original contract offered for the completion of that task. Consequently, the principal's value function after a failure is identical to that obtained before the failure occurred, i.e. V_i. Therefore, the principal solves

$$\max_{\{q_i, \overline{w}_i, \underline{w}_i\}} \left\{ -q_i \overline{w}_i - (1-q_i)\underline{w}_i + \delta\left[q_i V_{i+1} + (1-q_i)V_i\right]\right\}$$

subject to

$$q_i u(\overline{w}_i) + (1-q_i)u(\underline{w}_i) - \psi(q_i) \geq 0 \tag{9}$$

and

$$q_i^* \in \arg\max_{q_i} \left\{ q_i u(\overline{w}_i) + (1-q_i)u(\underline{w}_i) - \psi(q_i) \right\} \tag{10}$$

where again $V_{N+1} \equiv R$. Constraint (9) is just the participation constraint, while (10) is the agent's incentive compatibility constraint, requiring that the agent choose effort in order to maximize his expected utility. Instead of solving the problem under constraint (10), the first-order approach will be employed, replacing (10) by the constraint

$$u(\overline{w}_i) - u(\underline{w}_i) - \psi'(q_i) = 0 \tag{11}$$

This approach simplifies the analysis by replacing the continuum of constraints given by (10) with the constraint that the agent's utility be at a stationary point. As in Lambert (1983) and Spear and Srivastava (1987), it is hypothesized that the first-order approach is valid, noting the static nature of the problem and appealing to the analysis of Rogerson (1985).[5]

Denote by λ_i the multiplier on the participation constraint (9) and by μ_i the multiplier on the agent's first-order condition (11). Because of the principal's inability to commit, the constraints have to be satisfied attempt by attempt, task by task, as was the case in the first-best benchmark. Hence the subscripts on the multipliers. The optimal contract for the completion of task i, is characterized by the following set of equations:

$$\delta\left[V_{i+1} - V_i\right] - \left[\overline{w} - \underline{w}_i\right] + \lambda_i\left[u(\overline{w}) - u(\underline{w}_i) - \psi'(q_i)\right] - \mu_i\psi''(q_i) = 0 \tag{12}$$

$$q_i - q_i\lambda_i u'(\overline{w}_i) - \mu_i u'(\overline{w}_i) = 0 \tag{13}$$

$$-1 + q_i + (1-q_i)\lambda_i u'(\underline{w}_i) - \mu_i u'(\underline{w}_i) = 0 \tag{14}$$

$$\lambda_i\left[q_i u(\overline{w}_i) + (1-q_i)u(\underline{w}_i) - \psi(q_i)\right] = 0 \tag{15}$$

Equation (12) characterizes the optimal effort, (13)-(14) characterize the optimal wages in case of success and failure respectively, while (15) is the complementarity condition for the

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[5]The analysis of Pavoni (2004) suggests that the conjecture is indeed correct for the two-outcome case treated here. See also the analysis by Jewitt (1988) for a more general treatment of the first-order approach in static problems.

agent's participation constraint (9). Solving (13)-(14) simultaneously yields the following values for the multipliers:

$$\lambda_i = \frac{(1-q_i)u'(\overline{w}_i) + q_i u'(\underline{w}_i)}{u'(\underline{w}_i)u'(\overline{w}_i)} \tag{16}$$

$$\mu_i = \frac{q_i(1-q_i)\left[u'(\underline{w}_i) - u'(\overline{w}_i)\right]}{u'(\underline{w}_i)u'(\overline{w}_i)} \tag{17}$$

Clearly, λ_i is positive, so the agent's participation constraint (9) is binding. Furthermore, it follows from (11) that $\underline{w}_i < \overline{w}_i$ and thus μ_i is positive. Last, (11) implies that (12) reduces to

$$\delta\left[V_{i+1} - V_i\right] = \left[\overline{w}_i - \underline{w}_i\right] + \mu_i \psi''(q_i) \tag{18}$$

This shows that in providing incentives to the agent, the principal induces inefficiency in the effort choice, effectively driving a wedge between the principal's marginal benefit from success and his marginal cost in terms of transfers to the agent.[6] In contrast to the first-best contract, the agent is no longer fully insured, and his wage is increasing in outcome (i.e. he is rewarded more for success than for failure). Note however, that the offered contract does not allow for any intertemporal smoothing of wages, i.e. it is memoryless, a fact that derives from the principal's inability to commit. For example, in figure 1 the agent's continuation payoffs at point (5,5) are the same irrespective of wether the black or the grey path led the agent to that point.

To see how the offered contract evolves with progress, arguments paralleling those for the first-best scenario show that under spot contracting, induced effort is increasing in progress.[7] Since the participation constraint (9) is binding, increasing effort over the stages of the project means that the agent's expected wages are increasing in progress. Last, from (11) it also follows that the incentive schemes are increasingly high-powered as progress is made.

The comparative statics results on expected completion time $E[T_N]$ derived under the first-best benchmark carry over to the scenario with spot contracting. But, from revealed preferences, it follows that the sequence of implemented efforts is lower than the first-best sequence.

Summing up, the sequence of optimal spot contracts is similar to the first-best contract in all but one respect. While they still implement a sequence of effort and expected wages which is increasing in progress and is memoryless, the presence of asymmetric information induces suboptimal risk sharing. Since the agent is exposed to more risk than under the first-best, his effort choices are distorted downwards, creating higher expected delay and thus longer expected development time.

5.   Long-Term Contract

Now I consider a setting in which the principal can commit to any long-term contract. In all generality, the continuation contract offered after any attempt j on any task i may be

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[6]In a setup with a risk-neutral agent protected by limited liability, it is easily shown that even under asymmetric information, the agent's incentive compatibility constraint is not binding. Although the first-best level of effort is thus implemented, the agent's participation constraint is binding, thus distorting the wage transfers away from the first-best levels.

[7]To derive this result, the assumption that $\psi''' \geq 0$ is needed.

a function of the entire history of failures and successes on the current and all previous tasks. On the face of it, this history dependence enormously complicates the analysis and the characterization of the optimal long-term contract.

Fortunately, there is a very simple way of summarizing history and reducing the problem to a relatively straightforward dynamic programming problem. In a nutshell, the approach consists of introducing the agent's continuation utility as a random state variable. Therefore, the problem has two state variables, the agent's continuation utility and the current state of the project (i.e. the number of completed tasks). This allows for a recursive formulation of the principal's problem which is amenable to standard Kuhn-Tucker analysis. This approach was developed in Spear and Srivastava (1987) for an infinite horizon, continuous action, continuous outcome setup and expounded by Laffont and Martimort (2001) in an infinite horizon, binary action, binary outcome setup.

Reduction of the Problem. Since the principal is allowed to (and can credibly commit to) write long-term contracts, the problem at any time must explicitly include the effects that current data (and outcomes) have on the continuation of the contract. In the previous analysis, there were no such intertemporal links, and thus it was unnecessary to incorporate such effects in the notation, although those setups were of course special cases of the current setup.

First, define the agent's expected discounted utility after having failed $j-1$ attempts on task i as

$$U_{i,j} = \max_{q_{ij}} \left\{ -\psi(q_{ij}) + q_{ij} \left[u(\overline{w}_{ij}) + \delta U_{i+1,1} \right] + (1 - q_{ij}) \left[u(\underline{w}_{ij}) + \delta U_{i,j+1} \right] \right\} \qquad (19)$$

where $U_{N+1,1} \equiv 0$. In the setups considered in sections 3 and 4, the optimal contracts were memoryless, and so $U_{i,k+1} = U_{i,j+1}$ for all k, j. With commitment, promises of future rewards (or punishments) can be used to incentivize the agent in performing well on current tasks. Thus the current outcome may influence the future contracts faced by the agent, and this influence is reflected by the presence of the terms $U_{i+1,1}$ and $U_{i,j+1}$ in the agent's expected future utility. Thus U_{ij} is simply the agent's value function at a point where attempt j on task i is about to be made, given a set of continuation contracts offered by the principal.

Equivalently, the principal's value function, having promised the agent continuation value U_{ij}, is given by

$$V(U_{i,j}) = \max_{\left\{ q_{ij}, \overline{w}_{ij}, \underline{w}_{ij}, U_{i+1,1}, U_{i,j+1} \right\}} \left\{ q_{ij} \left[\delta V(U_{i+1,1}) - \overline{w}_{ij} \right] + (1 - q_{ij}) \left[\delta V(U_{i,j+1}) - \underline{w}_{ij} \right] \right\}$$

$$(20)$$

where $V(U_{N+1,1}) \equiv R$. More precisely, the time t value function of the principal could be written as $V(i, U(h_t))$, where i is the task currently being attempted and $U(h_t)$ is the level of promised utility after history h_t. In order to lighten notation, the i subscript on the agent's utility will keep track of the stage of the project and U_{ij} will refer to the promised level of utility given some history. It follows by the analysis of Spear and Srivastava (1987) that $V(\cdot)$ is decreasing and concave.[8]

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[8] This means that the principal's value is decreasing in the amount promised to the agent, and at an increasing rate.

The incentive compatibility constraint is now given by

$$q_{ij}^* \in \arg\max_{q_{ij}} \left\{ -\psi(q_{ij}) + q_{ij} \left[u(\overline{w}_{ij}) + \delta U_{i+1,1} \right] + (1 - q_{ij}) \left[u(\underline{w}_{ij}) + \delta U_{i,j+1} \right] \right\} \tag{21}$$

while the individual rationality (or intertemporal participation) constraint is given by

$$U_{1,1} \geq 0 \tag{22}$$

Constraint (22) derives from the fact that the agent commits to the long-term contract and consequently, there is only one intertemporal constraint to consider. Were the agent not to commit, the additional constraints $U_{i,j} \geq 0$ would have to be satisfied for all attempts j on all tasks i. This case will be discussed in section 6.

As in the previous sections, the first-order approach will be used, and the incentive compatibility constraint (21) is therefore replaced by the agent's first-order condition, given by

$$\psi'(q_{ij}) = \left[u(\overline{w}_{ij}) - u(\underline{w}_{ij}) \right] + \delta \left[U_{i+1,1} - U_{i,j+1} \right] \tag{23}$$

The principal's objective is then to solve the following problem:

$$\max_{\left\{ q_{ij}, \overline{w}_{ij}, \underline{w}_{ij}, U_{i+1,1}, U_{i,j+1} \right\}} \left\{ q_{ij} \left[\delta V(U_{i+1,1}) - \overline{w}_{ij} \right] + (1 - q_{ij}) \left[\delta V(U_{i,j+1}) - \underline{w}_{ij} \right] \right\}$$

subject to (22) and (23).

Characterization. First, I characterize how the optimal long-term contract rewards success, respectively punishes failure, on a given attempt j on a given task i. Letting λ be the multiplier on (22) and μ_{ij} those on constraints (23), the optimum is characterized by the following set of equations:

$$\delta \left[V(U_{i+1,1}) - V(U_{i,j+1}) \right] - \left[\overline{w}_{ij} - \underline{w}_{ij} \right] - \mu_{ij} \psi''(q_{ij}) = 0 \tag{24}$$

$$-q_{ij} + \mu_{ij} u'(\overline{w}_{ij}) + \lambda q_{ij} u'(\overline{w}_{ij}) = 0 \tag{25}$$

$$-(1 - q_{ij}) - \mu_{ij} u'(\underline{w}_{ij}) + \lambda(1 - q_{ij}) u'(\underline{w}_{ij}) = 0 \tag{26}$$

$$q_{ij} V'(U_{i+1,1}) + \mu_{ij} + \lambda q_{ij} = 0 \tag{27}$$

$$(1 - q_{ij}) V'(U_{i,j+1}) + \lambda(1 - q_{ij}) - \mu_{ij} = 0 \tag{28}$$

$$\lambda U_{1,1} = 0 \tag{29}$$

Equation (24), which characterizes the optimal effort, has been reduced by making use of the agent's first-order condition (23). Equations (25)-(26) characterize the optimal wages in case of success and failure respectively, while (27)-(28) characterize the corresponding continuation utilities. Last, (29) is the complementarity condition for the agent's participation constraint (22).

Solving (25)-(26) simultaneously yields the following expressions for the multipliers:

$$\lambda = \frac{(1 - q_{ij}) u'(\overline{w}_{ij}) + q_{ij} u'(\underline{w}_{ij})}{u'(\underline{w}_{ij}) u'(\overline{w}_{ij})} \tag{30}$$

$$\mu_{ij} = \frac{q_{ij}(1 - q_{ij}) \left[u'(\underline{w}_{ij}) - u'(\overline{w}_{ij}) \right]}{u'(\underline{w}_{ij}) u'(\overline{w}_{ij})} \tag{31}$$

Note the similarity between (30)-(31) and the multipliers (16)-(17) obtained under spot contracting. Apart from the subscripts, these are identical, but the constraints they multiply are of course different. It follows that both $\lambda > 0$ and $\mu_{ij} > 0$. Thus, the agent's intertemporal participation constraint (22) is binding.

Next, I characterize how the optimal long-term contract evolves with success and failure. From (25)-(27) and (26)-(28), the following equations are obtained:

$$V'(U_{i+1,1}) = \frac{-1}{u'(\overline{w}_{ij})}$$

$$V'(U_{i,j+1}) = \frac{-1}{u'(\underline{w}_{ij})}$$

Dividing the first equation by the second, yields the following relationship between wages and continuation utilities:

$$\frac{V'(U_{i+1,1})}{V'(U_{i,j+1})} = \frac{u'(\underline{w}_{ij})}{u'(\overline{w}_{ij})} \tag{32}$$

Condition (32) simply equates the principal's and the agent's marginal rates of substitution between current and future consumption for each state of nature.

Equations (27)-(28) give

$$\mu_{ij} = q_{ij}(1 - q_{ij})\left[V'(U_{i,j+1}) - V'(U_{i+1,1})\right]$$

which, equated with (31), yields

$$\frac{u'(\overline{w}_{ij}) - u'(\underline{w}_{ij})}{V'(U_{i+1,1}) - V'(U_{i,j+1})} = u'(\underline{w}_{ij})u'(\overline{w}_{ij}) > 0 \tag{33}$$

Since the right-hand side of this equation is positive, this implies that either $u'(\underline{w}_{ij}) > u'(\overline{w}_{ij})$ and $V'(U_{i,j+1}) > V'(U_{i+1,1})$, which in turn implies that $\underline{w}_{ij} < \overline{w}_{ij}$ and $U_{i,j+1} < U_{i+1,1}$, or $u'(\underline{w}_{ij}) < u'(\overline{w}_{ij})$ and $V'(U_{i,j+1}) < V'(U_{i+1,1})$, which in turn implies that $\underline{w}_{ij} > \overline{w}_{ij}$ and $U_{i,j+1} > U_{i+1,1}$. But only the first case is consistent with the agent's first-order condition (23).

Condition (33) is important. It means that in providing incentives, the principal rewards success not only by the current wages, but also by increasing the continuation payoffs following a success. Thus, the optimal long-term contract has the *memory property*. In other words, the optimal long-term contract is such that wages and future rewards are aligned and thus reinforce each other in providing incentives.

Milestones. The characterization thus far also shows that in the optimal contract, incentives are not only provided by changes in the expected continuation utilities. I.e., it shows that it is not the case that the agent is paid only at the end of the relationship when task N has been successfully completed. The agent must be paid wages along the way. Neither is it not optimal to provide incentives only through wages received upon overall project completion and letting there be optimal risk-sharing at intermediate stages. This is because of the insurance role provided by intertemporal smoothing of the rewards (Lambert, 1983, noted this feature in a finitely repeated moral hazard setup).

This also has implications for the optimal *milestone frequency*. According to Lichtenstein (2003),

> "A [...] development milestone is defined by four elements: a deliverable, a
> date when the deliverable is expected, a payment to be paid on acceptance of the
> deliverable, [and] damages to be paid in case of delay in providing the deliverable".

Milestones (or *progress payments*) are a very frequently used method for controlling progress on large-scale projects such as weapons systems or drug development. E.g., a biotech company may be paid one installment from its financier when a product receives approval from the Food and Drug Administration, and another when the final product is marketed. What the present analysis shows is that in the optimal long-term contract, it is not optimal to cluster tasks together in larger milestones, with correspondingly larger milestone payments, instead of paying a sequence of smaller amounts. That is to say, it is not optimal to write a contract stipulating payment after, say, every fourth task has been completed. Again, this derives from the agent's risk aversion, which makes consumption smoothing desirable.

Further Characterization. Continuing the characterization of the optimal long-term contract, solving (27)-(28) for λ yields

$$\lambda = -q_{ij}V'(U_{i+1,1}) - (1 - q_{ij})V'(U_{i,j+1})$$

But from the envelope theorem, it follows that

$$V'(U_{i,j}) = -\lambda$$

In turn, this implies that

$$V'(U_{i,j}) = q_{ij}V'(U_{i+1,1}) + (1 - q_{ij})V'(U_{i,j+1}) \tag{34}$$

This equation shows that in the optimal long-term contract, the principal intertemporally smoothes rewards to the point that his marginal disutility of promising one more unit of wealth to the agent today, is equal to the expected marginal disutility of promising the agent one more unit of wealth in the future. Laffont and Martimort (2001) term this the *martingale property*. Note that, since $V'(U_{i,j+1}) > V'(U_{i+1,1})$, the martingale property (34) also implies that

$$V'(U_{i+1,1}) < V'(U_{i,j}) < V'(U_{i,j+1})$$

But since $V(\cdot)$ is concave, these inequalities imply that

$$U_{i,j+1} < U_{i,j} < U_{i+1,1}$$

The first of these inequalities means that in the optimal long term contract, the agent's continuation utility is decreasing in the number of failed attempts. In other words, after a failure on a particular task, the agent is worse off than before the failed attempt was made. The second inequality means that the agent, after a successful attempt on a given task, is better off than before that successful attempt was made, even though the success means that the agent is rewarded for one task less in the continuation contract.

Summing up the characterization so far, the optimal long-term contract has the memory property, which means that in providing incentives, the principal uses both current wages

and future prospects. While there is not optimal risk sharing, as was the case too under spot contracting, memory serves in relaxing the incentive problem by providing the agent with some measure of insurance through intertemporal consumption smoothing.

A full characterization of the optimal long-term contract would include a characterization of the implemented effort at all points on the progress grid in figure 1, as well as a characterization of offered wages and continuation utilities at all those points. Unfortunately, the interaction between the problem's two state variables renders a much sharper analysis infeasible.[9] This is because in the optimal long-term contract, it is not necessarily the case that the sequence of implemented efforts is monotone in progress on the project. When this is the case on two consecutive tasks though, some further characterization can be made.

Suppose then, that the sequence of implemented efforts q_{ij} is increasing in progress i. Rewriting (34) yields

$$V'(U_{i,j}) - V'(U_{i,j+1}) = q_{ij} \left[V'(U_{i+1,1}) - V'(U_{i,j+1}) \right] \tag{35}$$

Then, the extent to which a failure is punished as progress is made, i.e. $U_{i,j} - U_{i,j+1}$ can be characterized as follows.

Note that for increasing effort, it follows from the agent's first-order condition (23) that the sum of the differences $\overline{w}_{ij} - \underline{w}_{ij}$ and $U_{i+1,1} - U_{i,j+1}$ must increase in i. From (32), it follows that these differences must, in the optimal contract, both increase, in order to maintain equality between the agent's and the principal's marginal rates of substitution between current and future compensation. In turn, this implies that the right-hand side of (35) must increase, and thus so must the left-hand side. It follows that $U_{i,j} - U_{i,j+1}$ must increase in progress i. The implication is as follows. The closer the project is to overall completion, the higher is the cost of an additional period's delay. In turn, this implies that the agent is punished exceedingly harder for failure as progress is made.

As mentioned above, it is not immediate that the principal wishes to induce an increasing sequence of efforts irrespective of history. It may be that after a particularly lucky streak, the agent has been promised so much in the continuation contract, that the principal finds it optimal to "regulate" the agent's rents by reducing the probability of further success.

Renegotiation. As a last point, it should be noted that the optimal long-term contract is renegotiation proof by construction. That is, the continuation contracts are such that the principal would not be tempted to renegotiate them, regardless of the outcome and history experienced to that point.

6. Constrained Contracts

So far, the analysis has concentrated on optimal contracts under constraints relating to either informational asymmetries or the principal's ability to commit. While these are certainly important from a practical perspective, reality often imposes additional constraints not belonging to either of those categories. In particular, legal or political considerations can further reduce the class of contracts that can be enforced. In this section, four such constraints are discussed.

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[9]For comparison, the setups considered by Spear and Srivastava (1987) and Laffont and Martimort (2001) have only one state variable, namely the agent's continuation utility.

Limited Liability Constraints. A common restriction in moral hazard situations is that wages paid to the agent cannot fall below some threshold, regardless of the outcome of the agent's efforts. In the present setup, such a constraint amounts to requiring that

$$w_{ij} \geq 0 \tag{36}$$

for all tasks i and all attempts j, regardless of whether there is success or failure. Adding this restriction essentially bounds the agent's utility $U_{i,j}$ from below. Since limited liability constraints are typically binding, the upshot is that they may make it more expensive for the principal to induce effort. Recall that incentives are provided by the variation in utility across states of nature. If the agent's rewards (or rather, punishments) are bounded below, the principal must raise rewards for success accordingly, in order to induce the agent to exert effort. Since the agent is risk averse, the principal may find it optimal to induce effort at a lower level than that induced in the absence of limited liability constraints. This will tend to increase the probability of failure and thereby delay the time of project completion.

Penalty Clauses and Liquidated Damages. A long-standing puzzle in contract law is that under the common law system, courts rarely uphold contracts with so-called penalty clauses. A penalty clause is a contractual provision that entitles the principal to impose penalties on the agent in case of breach of contract. In the current setup, a penalty clause is simply the feature of the optimal contract that stipulates that the agent's utility be reduced after a failure on a given task. Courts will, however, accept so-called liquidated damages, i.e. a transfer from the agent to the principal reflecting losses that the agent's unsatisfactory performance has caused the principal. The difference between penalties and liquidated damages is that the latter reflect only "real" or "reasonable" losses from the agent's non-fulfillment of the contract, while the former allows the principal to set the penalty at will. In the current setup, as is the case in most moral hazard models, the agent is rewarded for success and punished for failure in order to provide correct incentives ex ante. This may well involve threats of punishments that ex post do not reflect the true damage or loss imposed on the principal. But if it is known ex ante that the contract will not be upheld ex post, the principal essentially loses part of his ability to provide incentives through outcome linked wages. The upshot is that the quantities $[U_{i,j} - U_{i,j+1}]$ and $[\overline{w}_{ij} - \underline{w}_{ij}]$ are bounded. Thus, while the limited liability constraints (36) limit the agent's utility from below, inability to impose penalties limits the variation in the agent's utility directly. This may severely limit the principal's ability to induce timely project completion through contracting.

Budgetary Horizons. It has been argued that public procurement agencies should abstain (or be barred) from writing long-term contracts, or contracts covering more than one budget period at a time. As noted by Laffont and Tirole (1993, p. 5),

> "[...] regulatory contracts extending beyond some specified *time horizon* may be illegal. In the United States, electric utility regulators cannot sign binding long-term contracts with the firms they regulate. Also, there has been much debate about allowing the Department of Defence (DOD) to engage in multiyear procurement (i.e., to commit funds for a substantial part of or the complete project)."

The implication of such constraints is that the principal's ability to smoothe the agent's rents over time is reduced. This is because such smoothing is obtained in the optimal long-term contract by letting future compensation positively covary with current compensation. But if the time interval covered by the contract is reduced, so is the extent to which promises of future rewards (or punishments) can be used to incentivize the agent to exert effort on the task at hand. Clearly, the longer the budgetary horizon, the less impact does this constraint have. The worst case is actually the setting with horizon $h = 1$, which is just that of spot contracting analyzed in section 4.

Non-Commitment of Agent. To this point, the analysis has focused on the principal's ability to commit. Sometimes, it may be that the agent is unable to commit to any long-term contract, and must be incentivized to work by a series of shorter-term contracts. In terms of the problem studied here, limited commitment of the agent amounts to replacing constraint (22) with the constraints

$$U_{i,j} \geq 0 \tag{37}$$

for all tasks i and attempts j. Constraints (37) are clearly less stringent than the limited liability constraints (36), but have much the same qualitative effects.[10] While under agent risk neutrality, these constraints would just mean a transfer of wealth from principal to agent, under risk aversion these will typically induce lower effort and thus longer expected project duration.

7. CONCLUSION

In this paper, I have set out a model of project completion in which there is time to build with stochastic progress, and where an agent's progress-enhancing effort is subject to moral hazard. I found that while some delay may be inevitable, asymmetric information exacerbates the problem.

The main lessons of the analysis are as follows. First, incentives should adequately reflect the value of effort on (and the cost of delay of) the overall project. In particular, this involves making incentives increasingly high-powered as progress is made, reflecting the fact that project value is increasing in progress.[11] Second, the analysis shows that an important factor in determining expected time of project completion is the principal's ability to commit to long-term contracts. The longer the contracts the principal is able to credibly commit to, the easier it becomes to provide the agent with correct incentives. Last, a number of constraints often observed in practice, such as limited liability constraints, non-commitment of the agent and non-enforcement of penalty clauses, imply that even if the principal is able to commit to long-term contracts, he may in practice be severely restricted in his ability to provide incentives. While it is probably beyond the ability of most principals to do away with such constraints, the analysis shows that properly written contracts that take incentive considerations into account may go a long way in reducing inefficient delay of long-term projects.

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[10] Constraint (22) does not rule out the possibility that after unsatisfactory performance, the agent's ex post continuation utility becomes negative.

[11] In the optimal long-term contract, this statement is true when controlling for the level of the agent's continuation utility.

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