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INCENTIVES AND DISCRIMINATION

by

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Incentives and Discrimination[1]

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ABSTRACT: Optimal incentive mechanisms may require that agents are rewarded differentially even when they are completely identical and are induced to act the same. We demonstrate this point by means of a simple incentive model where agents' decisions about effort exertion is mapped into a probability that the project will succeed. We give necessary and sufficient conditions for optimal incentive mechanisms to be discriminatory. We also show that full discrimination across all agents is required if and only if the technology has increasing return to scale. In the non-symmetric framework we show that negligible differences in agents' attributes may result in major differences in rewards in the unique optimal mechanism.

1. Introduction

The tension between efficiency and equality in incentive schemes is an issue which is often debated in organizations. The notion that benefits should be assigned to individuals in a non-uniform manner that takes into account qualifications and performances is well established. Yet it is sometimes claimed that favoritism and discrimination often lead to differential rewards even when individuals do not differ significantly in their attributes, a phenomenon which is regarded as counter-efficient. The purpose of this paper is to argue that, from the point of view of optimal incentives, differential rewards may be unavoidable even when individuals are completely identical and when the mechanism aims at inducing all agents to exert effort.

The fact that optimal incentive mechanisms may require non-symmetric rewards even when agents are identical in their qualifications may not be surprising in some environments. If agents, for example, are asymmetrically informed about each other's exertion of effort, it may require different levels of incentives to induce each of them to exert effort. In this case optimal mechanisms are expected to yield non-symmetric

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rewards that depend on the information that agents have about each other (see Winter, 2001, for a full analysis of such a model). Another case in which non-symmetric mechanisms seem trivially unavoidable is when the principal's objective is to induce some but not all agents to exert effort (for example, when contributions by only a subset of agents are sufficient to guarantee the project's success). In this case it will be wasteful to reward all agents equally. Promising positive rewards (contingent on the success of the project) to some and zero to others seems to be the optimal incentive scheme.

What we find more surprising is that optimal mechanisms may have to be discriminatory even when all agents are completely identical (in terms of all their characteristics including information) and when the objective of the mechanism is to induce all of the agents to exert effort. We demonstrate our point with a simple model of organization similar to the one used in Winter (2001) in the context of optimal allocation of responsibility in hierarchical organizations.

A project is managed by n agents each of which is responsible for a different task. If an agent exerts effort in performing his task he increases the probability that his task will end successfully from α to 1 at a cost c, which is constant across agents. The overall project succeeds only when all tasks end successfully. Neither the principal nor the agents themselves can observe each other's effort. Therefore, the mechanism must reward agents only as a function of whether the project ends successfully or not. An optimal mechanism induces agents to exert effort in any Nash equilibrium, and it does so at a minimum total reward.

Our first observation in Proposition 1 is that any optimal mechanism must treat all agents differently. We then extend the framework in Section 3 by referring to general success technologies that map agents' decisions to a probability of the project's success. We provide a necessary and sufficient condition for the existence of symmetric mechanisms indicating that the nonexistence of symmetric mechanisms is more general than might appear in our benchmark model.

The intuition behind the fact that optimal mechanisms may have to be non-symmetric is quite simple. If agents' exertion of effort induces a positive externality on the effectiveness of other agents' effort, it is optimal to promise high rewards to some agents so as to make the others confidently believe that these highly paid agents will contribute, hence allowing the planner to save resources by offering other agents substantially less. Invoking this argument iteratively shows that when the project technology involves increasing returns to scale, no two agents should earn the same reward in spite of the fact that all agents are identical. In fact the optimal mechanisms give rise to an endogenous hierarchy of incentives[3] at the top of which one agent is induced to exert effort regardless of his beliefs about other agents' actions, and at the bottom of which one agent is provided sufficient incentive to exert effort only when he believes that all the rest will do so as well. Interestingly, the property of increasing returns to scale is not only a sufficient condition for full discrimination. We also show that it is also a necessary condition -- if it fails to hold, some agents must be paid identically at any optimal mechanism. In Section 5 we address the non-symmetric case to point-out that discrimination plays a role even when the optimal mechanism is unique. We show that under increasing returns to scale negligible differences in agents characteristics may result in major payment differences in the now unique optimal mechanism. We provide the analysis for the case of differential effort costs as well as the case of differential probabilities of success under no investment. However, in contrast to the symmetric case in which the allocation of agents and tasks to the different levels of the incentive hierarchy mentioned above was arbitrary, now this allocation is uniquely determined. Specifically, agents with low effort costs are assigned to higher levels of the incentive hierarchy and tasks which are more sensitive to the effort decision are assigned to higher levels as well.

Finally, in Section 6 we consider environments in which agents can make credible commitments about effort exertion. We model it by simply changing the solution concept for implementation - from Nash equilibrium to Strong Nash equilibrium (see Maskin, 1979). This latter concept takes into account not only unilateral deviations but also deviations by groups. We show that in such environments the optimal mechanism is symmetric for any monotonically increasing technology.

[3] Hence our model can offer a explanation for the use of ranks in organizations, which differs from other well known models in Labor Theory including models of long term incentives (e.g. Lazear 1979) or Tournament models (Lazear and Rosen 1981).

In our model the effort decisions by some agents affect other agents' incentives to exert effort. To this extent this paper is related to the literature on network externalities e.g., Farrell and Saloner (1985) Katz and Shapiro (1986). The intuition behind the advantage of discrimination in our incentive mechanisms is related to the idea of price discrimination by monopolies producing goods with positive consumption externalities, for example in the form of introductory pricing (see for example Bensaid and Lesne (1996) and Cabral, Salant and Woroch (1999)). It is also related to Segal's (2001) general model of trade contracts where he observed that when trade generates positive externalities on other agents, the principal gains by discriminating whenever he wants to sustain his preferred trade as a unique equilibrium. However the incentive model here differs from those used in the literature above in several features. The most important of them is the fact that agents' effort decision are unobservable. Hence contracts in our model are allowed to make contingencies only on the final outcome of the project (and not on the actual "trade" chosen by the agent). Furthermore, our framework allows the establishment of increasing returns to scale as a necessary condition for full discrimination and addresses other organizational issues in the non-symmetric case.

2. The Model

The organizational project involves n tasks each performed by a different individual (agent). Each agent has to decide whether to exert effort towards the performance of his task or not. The cost of effort is c and is constant across all players. We henceforth use the term investment for the action of exerting effort.

If an agent invests, his task ends successfully with probability 1. If he doesn't invest, the probability of his task ending successfully is only α, which is again constant across all agents. Agents are not informed about each other's investment decisions. Thus these decisions will be modeled as if made simultaneously.

For our benchmark model we will assume that the project as a whole ends successfully if and only if all the tasks are preformed successfully. We will later consider a general class of technologies that do not necessarily have this "O-Ring"[4] property.

Since the effort decision of an agent is unobservable to the rest of the players and the principal agents' rewards in the mechanism depend only on whether the project ends successfully or not. Specifically, if the project fails all agents receive a zero reward, but if it succeeds they are paid the rewards $v = (v_1,...,v_n)$. We identify here the vector v with the incentive mechanism. We note that each such mechanism gives rise to a normal form game $G(v)$ in which each player i has two strategies: $d_i = 1$ for investment and $d_i = 0$ for non-investment. The payoff function of the game is given as follows: For a strategy combination $d = (d_1,\ldots,d_n) \in \{0,1\}^n$ the payoff function for player i, is given as follows: $f_i(d) = v_i\alpha^{s(d)} - c$ if $d_i = 1$ and $f_i(d) = v_i\alpha^{s(d)}$ if $d_i = 0$, where $s(d) = |\{j|\ d_j = 0\}|$ is the number of individuals who choose to shirk.

We say that a mechanism v is incentive-inducing *(INI)* if v induces all players to invest in every equilibrium, i.e., $d = (1,1,...,1)$ is the only Nash equilibrium of the game $G(v)$.

We will say that a mechanism v is an optimal *INI* if it minimizes the total reward among all *INI* mechanisms[5].

Proposition 1 claims that in any optimal *INI* mechanism no two players are rewarded equally.

For a permutation θ on the set of agents, and a vector x, we denote by $\theta(x)$ the vector with $\theta_i(x) = x_{\theta(i)}$.

Proposition 1: Let $v^* = (c/(1-\alpha), c/\alpha(1-\alpha), ..., c/\alpha^{n-1}(1-\alpha))$. A mechanism v is an optimal *INI* mechanism if and only if $v = \theta(v^*)$ for some permutation θ.

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[4] See Kremer (1993)

[5] We will also allow the reward-minimizing mechanism not to be INI in itself. The formal definition should be the following: v is an optimal INI mechanism if (1) there exists no INI mechanism with less total reward and (2) for any ε $\{v_i+\varepsilon\}_{i\in N}$ is an INI mechanism. This technical caveat is innocent and is needed because rewards take continuous values.

Proof: We first note that $\theta(v^*)$ is an *INI* mechanism. Since all agents are symmetric we will denote by $v(k)$ the reward that would make an agent indifferent between investing and shirking given that he believes that exactly k other agents are investing, where $0 \leq k < n$. Note that if i chooses to invest his expected payoff is $v(k)\alpha^{n-k-1} - c$, whereas if he chooses not to invest the expected payoff is $v(k)\alpha^{n-k}$. Hence, $v(k)$ satisfies $v(k)\alpha^{n-k-1} - c = v(k)\alpha^{n-k}$, or $v(k) = c/\alpha^{n-k-1}(1-\alpha)$. We can therefore set $v^* = (v(n-1),v(n-2),...,v(0))$. Note also that $v(k) > v(k+1)$ and in particular $\theta_i(v^*) \geq v(n-1)$, which means that $d = (1, ... , 1)$ is a Nash equilibrium of $G(\theta(v^*))$. Hence, to show that $\theta(v^*)$ is an *INI* mechanism, it is sufficient to show that no equilibrium exists in which some group of agents shirks if we increase rewards by an arbitrarily small amount. Consider a strategy combination in which exactly k agents choose to invest where $0 \leq k < n$. Consider the players who are assigned the rewards $v(0), ... ,v(k)$. Any arbitrarily small increase in these rewards will make each of these players better off investing if one believes that k other players are investing as well. Hence, by an arbitrarily small increase of rewards beyond $\theta(v^*)$ we get $d = (1, ..., 1)$ as the unique equilibrium. We now have to show that $\theta(v^*)$ is optimal. We assume without loss of generality that θ is the identity permutation. Consider a mechanism u such that $u_i < v_i^*$ for some players and $u_j = v_j^*$ for the rest. Let r be the largest index for which $u_r < v_r^*$; then there is an equilibrium of $G(u)$ in which players $1,2, ..., r$ shirk and $r+1,...,n$ invest and this equilibrium survives for a sufficiently small increase of the rewards beyond u. This equilibrium will cease to exist only if we increase the reward of one of the players in $1,2, ... , r$ to become at least v_r^*. Hence, no *INI* mechanism can have a total reward which is less than $\sum_i v_i^*$.

3. General Success Technologies

In our benchmark model, agents' investments were mapped into a probability of the project's success in a particular way. In this section we will argue that the lack of symmetric optimal incentive mechanisms is also obtained in a more general framework. To this end we will view the project's technology as a function p from the set investment strategy profiles $\{0,1\}^N$ to $[0,1]$ specifying the probability of success for any given profile. Since our interest lies with the case in which all agents are identical, we will define a symmetric technology as a function $p: \{0,1,2,...,n\} \rightarrow [0,1]$ which gives the probability of the project's success as a function of the number of agents who choose to invest. We assume that extra investment always raises the probability of success, i.e., p is strictly increasing. Finally, as before, we assume that the principal is interested in inducing all agents to invest. The definition of an optimal *INI* mechanism remains the same.

A mechanism v is said to be symmetric if it assigns the same reward to all agents. Proposition 2 asserts that a necessary and sufficient condition for symmetry is that the "last" agent's marginal contribution to the project's success cannot exceed that of any other player. This condition implies a certain degree of substitution between the agents.

<u>Proposition 2</u>: A symmetric *INI* mechanism exists if and only if $p(n) - p(n-1) \leq p(k+1) - p(k)$ for all $0 \leq k < n-1$.

<u>Proof:</u> Consider again the reward $v(k)$ for which an agent is indifferent between investing and shirking if he believes that exactly k other agents are investing. With a general technology p a player's expected reward if he invests is $v(k)p(k+1) - c$, and with no investment it is $v(k)p(k)$. Thus $v(k)$ solves $p(k)v(k) = p(k+1)v(k) - c$ or $v(k) = c/[p(k+1)-p(k)]$. The condition in Proposition 2 implies that $v(n-1) \geq v(k)$ for all $k < n-1$. Consider now the mechanism $v = (v(n-1), ... , v(n-1))$. For this mechanism $d = (1, ... ,1)$ is a Nash equilibrium and for any arbitrary small increase of rewards, it is also the unique Nash equilibrium. Furthermore, if we decrease the reward for any agent $d = (1, ... , 1)$ is no longer not an equilibrium. Hence, v is a symmetric optimal *INI* mechanism. We now show that the condition of the proposition is necessary: suppose by way of contradiction that $p(n-1) - p(n) > p(k+1) - p(k)$ for some $k < n-1$ and that a symmetric optimal *INI* exists in which $v_j \equiv u$. Clearly u must be one of the values $(v(0),v(1),v(2), ... ,v(n-1))$. By the definition of $v(k)$ we have $v(n-1) < v(k)$ for some $k < n-1$. Let $k^* = argmax_k v(k)$. We first assume that $u < v(k^*)$. Note that in order to sustain investment by all players we must have $u \geq v(n-1)$. Since $v(n-1) \leq u < v(k^*)$, there must exist some $k \leq n-2$ such *that*

$v(k\text{-}1) \leq u < v(k)$. We now claim that under the mechanism u there exists a Nash equilibrium in which k agents invest and $n\text{-}k$ agents shirk. Indeed, no shirking player can profit by deviating to investment as investment requires a greater incentive given by the reward $v(k)$. Furthermore, no investing player will deviate by shirking because $v(k\text{-}1) \leq u$ and so any arbitrarily small increase of rewards results in agents preferring investment when $k\text{-}1$ other invest. Hence, u is not an *INI* mechanism as it yields equilibria in which some agents shirk. We now consider the case in which $u = v(k^*)$. Indeed, such a u is an *INI* mechanism. This is because $v(k^*)$ implies that $d = (1, \ldots ,1)$ is an equilibrium and $v(k^*) \geq v(k)$ for all k implies that no other equilibrium exists. However, the mechanism $u^* = (v(n\text{-}1),v(k^*), \ldots ,v(k^*))$ is an *INI* mechanism as well. Since $v(k^*) > v(n\text{-}1)$ this mechanism involves a smaller reward for the first agent and the same for the rest. Hence, u is not an optimal *INI* and we obtain the desired contradiction.

4. Increasing Returns to Scale

In this section we characterize the technologies under which the optimal mechanism rewards all agents differently. We show that this property is equivalent to the technology having increasing returns to scale. More specifically, we say that an *INI* mechanism v is fully discriminating if $v_i \neq v_j$ for every pair of agents i,j. We say that the technology p has increasing returns to scale if $D(k) = p(k+1) - p(k)$ $(k=0, \ldots ,n\text{-}1)$ is increasing in k.

<u>Proposition 3:</u> The technology p has increasing returns to scale if and only if all optimal *INI* mechanisms are fully discriminating.

<u>Proof:</u> If p has increasing returns to scale then $v(k) = c/[p(k+1) - p(k)]$ and $v(k)$ is decreasing in k. Hence, using the same argument as in the proof of Proposition 1, we obtain that the optimal mechanisms are given by $\theta(v(0), \ldots ,v(n\text{-}1))$ where θ is some permutation of the set of agents. Hence, all mechanisms are fully discriminatory. We now show that if all optimal *INI* mechanisms are fully discriminating, then p must have increasing returns to scale. As argued earlier, the payoffs in an *INI* mechanism must

involve only the values $v(0), \ldots ,v(n\text{-}1)$, so by assumption all these values are distinct. Consider an optimal *INI* mechanism and assume the following order of the values $v(k)$:
$v(k_0) > v(k_1) > \ldots > v(k_{n-1})$ (where $k_0, k_1, \ldots k_{n-1}$ is some permutation of $1,2,3 \ldots ,n$). We first note that $v(k_{n-1}) = v(n\text{-}1)$. Otherwise, there is some k_j with $v(k_j) < v(n\text{-}1)$ and $d = (1, \ldots, 1)$ cannot be a Nash equilibrium of the game because the player receiving $v(k_j)$ is better off deviating by shirking. We now establish by induction that $v(j) \geq v(k_j)$ for $j = 0,1, \ldots ,n\text{-}2$. First, $v(k_0) \geq v(0)$ by the definition of $v(k_0)$ as the largest among the values.

Now assume by induction that $v(j) \leq v(k_j)$ for all $j \leq r\text{-}1 < n\text{-}2$ and consider $j = r$. Suppose by way of contradiction that $v(r) > v(k_r)$. We argue that the mechanism admits a Nash equilibrium in which r agents invest and n-r shirk. Indeed, consider the set of agents whose payoffs are $v(k_0), \ldots ,v(k_{r-1})$. Call this set $R= \{0,1,\ldots,r\text{-}1\}$. Using iterative elimination of dominated strategies with the inductive hypothesis, we obtain that none of the agents in R shirk in any Nash equilibrium. This is done as follows: for player 0 who receives $v(k_0)$ investing is a dominant strategy. Given that player 0 invests, it is a dominant strategy for player 1 who receives $v(k_1)$ to invest as well. Continuing in this manner the result is that all players in R invest. It is therefore sufficient to argue that when the agents in R invest, no other agent can increase his payoff by shifting from shirking to investing. But this follows from the fact that for every player j in $N \backslash R$ the reward v_j satisfies $v_j \leq v(k_r) < v(r)$. We thus obtained that $v(j) \leq v(k_j)$ for $0 \leq j \leq n\text{-}1$. But since the sets $\{v(k_j);\ 0 \leq j \leq n\text{-}1\}$ and $\{v(j);\ 0 \leq j \leq n\text{-}1\}$ are identical – these inequalities must imply equality. Hence, we have $v(0) > v(1) > v(2) >, \ldots , > v(n\text{-}1)$. But as we argued at the beginning of the proof, the $v(j)$'s are the inverse of the $D(j) = p(j+1) - p(j)$, which is therefore increasing. Hence, p has increasing returns to scale.

5. The Non-symmetric Case

We now turn to the issue of discrimination in the non-symmetric case by pointing out that slight differences in agents' characteristics may result with major differences in rewards even when the optimal mechanism is unique. This is done by allowing either the effort costs or agents' probabilities of success to vary across agents. For the case of differential effort costs we show that under increasing returns the optimal mechanism is unique. Furthermore, if agents' effort costs are sufficiently close to each other it prescribes higher payments to agents of lower effort cost. However, the more interesting implication of this observation is the fact that agents who differ only slightly in their cost of effort may end up with major differences in rewards. This is a direct consequence of the fact that an agent's payoff in the optimal mechanism does not only depend on his own cost of effort (or skills) but also on the way he ranks relative to others. A similar finding applies when we consider differential probabilities of success in the benchmark model discussed in Section 2. In this case we show that agents with lower α_i are paid more in the unique optimal mechanism. In the sequel we will argue that these results can be interpreted as explaining the role of hierarchies as a coordination device[6].

<u>Proposition 4:</u> Let p be an increasing returns to scale technology and let $c_1 < c_2 <, \dots ,< c_n$ denote agents' effort costs, then the (unique) optimal mechanism pays player j
$v_j^* = c_j/[p(j) - p(j-1)]$, i.e., v_j^*/c_j is decreasing with j.

Note that the denominator of v_j^* depends only on how the agents' effort costs are ordered and not on their actual values. Hence, a slight difference in these values may result with major differences in the optimal rewards.

<u>Proof of Proposition 4</u>: Take any order θ: $N \rightarrow N$ and consider the mechanism v^θ with $v_j^\theta = c_j/[p(\theta(j)) - p(\theta(j-1))]$, i.e., the payoff for each agent is his effort cost divided by his marginal contribution with respect to some coalition size. Denote by $d_j^\theta = p(\theta(j)) - p(\theta(j-1))$ the payment per unit of cost for agent j. Setting up the incentive compatible equations as in the proof of Proposition 2 we find that any such mechanism is incentive-inducing (sustaining effort by iterative elimination of dominated strategies). Consider some agent for which $d_j^\theta = p(k+1) - p(k)$, $k = 0,1, \dots ,n-1$. Suppose that we drop j's payoff by ε without changing the payoffs of the rest, then there will exist an equilibrium in which all

[6] This of course in addition to other roles of hierarchies like authority (see Aghion and Tirole (1997)) or leadership (see Hermalin (1998))

agents i with $d_i^\theta \geq d_j^\theta$ shirk and the rest exert effort. To eliminate this equilibrium we now consider transferring this ε to a different agent i for which $d_i^\theta > d_j^\theta$. For sufficiently small ε such a transfer will not change the incentive of i, and the same equilibrium will prevail. The change in incentive will occur when $c_i/d_i^\theta + \varepsilon \geq c_i/d_j^\theta$ in which case i contributes if he believes that he is in a group of j contributors. But for such ε, we have: $\varepsilon \geq c_i/d_j^\theta - c_i/d_i^\theta$, and agent j's remaining payoff will be not more than $c_j/d_j^\theta - c_i/d_j^\theta + c_i/d_i^\theta$. Since $c_j < c_i$ and $d_i^\theta > d_j^\theta$ this remaining payoff is strictly less than c_j/d_i^θ. So j is not provided enough incentive to contribute when he believes that he is in a group of i contributors (replacing the role of agent i). This shows that the only candidates for *INI* in the non-symmetric case are the *n!* reward vectors described at the beginning of the proof. To notice that v^* is the least expensive among them, note that if $c_i > c_j$ and $d_i^\theta > d_j^\theta$, then changing the order by flipping the roles of i and j (i.e., paying j the reward c_i/d_j^θ and paying i the reward c_j/d_i^θ) without affecting the payoff of others, gets us an *INI* mechanism of a lower cost.

We now turn to the case of differential probabilities of success providing our analysis for the benchmark model discussed in Section 2.

<u>Proposition 5:</u> Consider the benchmark model in Section 2 and assume that $\alpha_1 < \alpha_2 <, \ldots, < \alpha_n$ and that c is the constant effort cost. Then the optimal mechanism is unique and is given by $v_i = \dfrac{c}{\prod_{j=i+1}^{n} \alpha_j (1-\alpha_i)}$ for $i < n$ and $v_n = \dfrac{c}{1-\alpha_n}$. Furthermore, negligible differences in the values of α_j's result in major differences in rewards.

We have seen that the optimal *INI* mechanisms generate hierarchy of incentives in the sense that different players have different stakes in the success of the project: agent *1* is induced to exert effort regardless of other agents' decisions whereas agent n will do so only if he believe that all the rest will exert effort as well. We submit that this observation can explain how hierarchies may emerge in organizations even when agents of different levels deal with similar tasks and when authority plays little role. Such hierarchies serve as a coordination tool by which each agent is guaranteed that those placed above him in the hierarchy will indeed exert effort -– providing sufficient incentive for him to do so as well. We point out that when using this interpretation of hierarchies, Propositions 4 and 5 tell us how hierarchies are endogenously determined when individuals or tasks differ in their attributes. Specifically, Proposition 4 implies that if agents differ in their cost of effort, then agents with low costs should be assigned to higher levels of the hierarchy. Furthermore, Proposition 5 asserts that if tasks differ in the probability of success under no investment (i.e., the probability α_i) then tasks with low α_i (those which are more sensitive to the effort decision) should be assigned to higher hierarchy levels as well.

<u>Proof of Proposition 5:</u> Take any order $\theta = i_1, i_2, \ldots, i_n$ and consider the mechanism v^{θ} given by

$v^{\theta}_{i_j} = \dfrac{c}{\prod_{k=j+1}^{n} \alpha_{i_k} (1-\alpha_{i_j})}$ for $j < n$ and $v^{\theta}_{i_n} = \dfrac{c}{(1-\alpha_{i_n})}$. We first claim that any such mechanism (and there are $n!$ of them) is incentive inducing. For that we will show that with these rewards exerting effort emerges as a unique Nash equilibrium with iterative elimination of dominated strategies. Consider first player i_1 if this player chooses 1 and the rest shirk, then the probability of success is $\prod_{k=2}^{n} \alpha_{i_k}$ while it is $\prod_{k=1}^{n} \alpha_{i_k}$ if he chooses 0. Hence if all other agents shirk, $v^{\theta}_{i_1}$ renders i_1 indifferent between shirking and exerting effort. This means that choosing effort is a dominant strategy for i_1 under $v^{\theta}_{i_1}$. Assume now that iterative elimination implies that $i_1, i_2, \ldots, i_{j-1}$ all exert effort then if i_j chooses 1 and $i_{j+1}, i_{j+2}, \ldots, i_n$ all choose 0, the success probability is $\prod_{k=j+1}^{n} \alpha_{i_k}$, and it is $\prod_{k=j}^{n} \alpha_{i_k}$ if i_j chooses 0 under the same conditions. Since $v^{\theta}_{i_j}$ solves the equation $v \prod_{k=j+1}^{n} \alpha_{i_k} - c = v \prod_{k=j}^{n} \alpha_{i_k}$, player i_j is indifferent between exerting effort and shirking under the specified condition which allows us to eliminate $d_{ij} = 0$. We now argue that the optimal *INI* mechanism must be v^{θ} for some order θ of the players. Consider such order θ. If we pay some player i_j $v^{\theta}_{i_j} - \varepsilon$ for a positive ε without changing the rewards of other players, then the game will have an equilibrium in which $i_1, i_2, \ldots, i_j$ shirk and the rest exert effort. This equilibrium will survive when transferring the ε to another player i_k as long as ε is small enough. Consider the minimal ε for which this equilibrium fails to survive, then $(v^{\theta}_{i_1},\ldots,v^{\theta}_{i_k}-\varepsilon,\ldots,v^{\theta}_{i_j}+\varepsilon, v^{\theta}_{i_n}) = v^{\theta'}$, where θ' is the order in which i_k and i_j exchange positions. A similar argument is given in more detail in the proof of Proposition 4. Finally, we argue that among the mechanisms v^{θ}, the optimal one corresponds to θ being the identity order and thus v^{θ} corresponds to the rewards specified in the statement of the proposition. For that we have to show that $\sum_{j=1}^{n} v^{\theta}_{i_j}$ is minimal when θ is the identity. Let θ_1 be the identity and θ_2 be the order in which i and $i+1$ exchange positions, i.e., $\theta_2 = (1,2, \ldots, i-1, i+1, i, i+2, \ldots ,n)$. It is enough to show that $v_i^{\theta_1} + v_{i+1}^{\theta_1} < v_i^{\theta_2} + v_{i+1}^{\theta_2}$.

Indeed, $v_i^{\theta_1} = \dfrac{c}{\prod_{j=i+1}^{n} \alpha_j (1-\alpha_i)}, v_{i+1}^{\theta_1} = \dfrac{c}{\prod_{j=i+2}^{n} \alpha_j (1-\alpha_{i+1})}$ and $v_i^{\theta_1} + v_{i+1}^{\theta_1} = \dfrac{1-\alpha_i\alpha_{i+1}}{\alpha_{i+1}(1-\alpha_i)(1-\alpha_{i+1})}$,

whereas $v_i^{\theta_2} = \dfrac{c}{\prod_{j=i+2}^{n} \alpha_j (1-\alpha_i)}, v_{i+1}^{\theta_2} = \dfrac{c}{\prod_{j=i+2}^{n} \alpha_j (1-\alpha_{i+1})\alpha_i}$ and $v_i^{\theta_2} + v_{i+1}^{\theta_2} = \dfrac{1-\alpha_i\alpha_{i+1}}{\alpha_i(1-\alpha_i)(1-\alpha_{i+1})}$.

The result now follows from the fact that $\alpha_i > \alpha_{i+1}$. Finally, note that when the probabilities of success are arbitrarily close to each other agents' rewards in the unique optimal mechanism are significantly different. Furthermore, under these conditions agents with lower α_i are paid more.

5. Commitment

If agents can make enforceable commitments to each other concerning investment and coordinate actions, then the principal can implement investment at less expense. In this case he can even achieve it with a symmetric mechanism. This is due to the fact that the possibility of agents to coordinate joint deviations filters out the "bad" equilibria in which only a subgroup of agents exerts effort. We analyze this framework by adopting the very same model but assuming that investment is implemented via *strong* equilibria (see also Maskin (1979) for strong equilibria implementation of social choice correspondences).

A strategy profile σ is a strong equilibrium if there exists no coalition of players S and a strategy profile $\sigma_S' = \{\sigma_i'\}_{i \in N}$ for that coalition such that all players in S are made better off by deviating to σ_S' assuming that players in $N \backslash S$ are still playing σ.

A mechanism v is an incentive-inducing mechanism via strong equilibria (*INIS*) if $d = (1, \ldots ,1)$ is the unique strong equilibrium of the investment game. Optimality is now defined in the same way as before.

We point out that the choice of solution concept for implementation should reflect the principal's assessment regarding the nature of interaction that takes place within the organizational environment. If the principal believes that the environment is sufficiently open and conducive to cooperation then the concept of strong equilibrium may be the appropriate one for implementation. Otherwise, only the standard Nash implementation is viable, requiring the extra burden of higher rewards and the necessity of discrimination.

Proposition 6: If the technology p is increasing,[7] then the unique optimal *INIS* mechanism is given by $v_j = c/[p(n) - p(n-1)]$ for all j.

Proof: We show the following: (1) with $v_j \equiv v = c/[p(n) - p(n-1)]$ the strategy profile $d = (1, \ldots ,1)$ is a strong equilibrium. In this stipulated equilibrium a player earns $vp(n) -c$. If a group of agents S of size s chooses to shirk, each of its members will earn $vp(n-s)$, which is less than $vp(n) -c$ since $p(n) - p(n-s) > p(n) - p(n-1)$. Using the same inequalities we have: (2) $d = (1, \ldots ,1)$ is the unique strong equilibrium. This is because any strategy combination in which s agents shirk has a profitable joint deviation for these players which is to invest[8]. Finally, (3) v as specified above is optimal. For this note that if some player's reward is less than $v = c/[p(n) - p(n-1)]$, then $d = (1, \ldots ,1)$ is not a Nash

[7] Not necessarily increasing returns to scale.
[8] If s = 1, then we have indifference, but if rewards increase by an arbitrarily small increment, this indifference is broken also for s =1 (see footnote 2).

equilibrium (and therefore not a strong equilibrium) since this player will choose to shirk when the rest invest getting $vp(n-1)$ instead of $vp(n) - c$.

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