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RISK AVERSION IN THE TALMUD

by

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Risk Aversion in the Talmud^{*†}

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Abstract

Evidence is adduced that the sages of the ancient Babylonian Talmud, as well as some of the medieval commentators thereon, were well aware of sophisticated concepts of modern theories of risk-bearing.

“If an evil witness arise against a man, to testify against him ... and the judges investigate the matter, and find that ... his testimony ... is a lie, then shall ye do unto him as he plotted to do unto his brother, and eradicate evil from your midst” (Deutoronomy 19, 16-19).

In Jewish law, a perjurer is treated exactly like the person against whom he testified would have been treated, had the perjury not been discovered. This rule is universal; it applies to the most trivial civil claim as well as to capital cases. Thus if Adams falsely testifies that Brown committed a capital crime, then Adams is executed; and if Adams falsely testifies that Brown owes Cox \$100, then Adams must pay Cox \$100.

^{*}Presented at the Institute for Mathematical Studies in the Social Sciences—Economics, Stanford University, August 4, 1981. Subsequent to that presentation, the author’s attention was drawn to an article by Zvi Ilani, “Models in the Economics of Uncertainty: The Cost of Concluding a Conditional Contract, according to the Talmud and the Halachic Literature,” *Iyunim Bekalkala (Investigations in Economics)*, The Israel Association for Economics, Jerusalem, Nissan 5740 (April 1980), 246-261 (in Hebrew). Inter alia, Ilani treats the Talmudic passage that forms the subject of this paper, and provides a fairly comprehensive review of the medieval commentaries thereon; undoubtedly, he was the first to recognize in print the relevance of this passage to modern economic theories of uncertainty. It is not clear, though, whether or not his understanding of the passage agrees with ours.

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There are, however, instances in which it is not a priori clear how the rule should be applied. Consider the following passage from the Babylonian Talmud¹:

“ ‘We testify² that John Doe divorced his wife and did not pay [the amount stipulated in] her marriage contract’³ [when in fact, he did not divorce her]. But [in punishing the perjurers, one should take into account that] he may eventually have to pay anyway!” (Makkoth 3a, in the Mishna⁴ in the middle of the page; the material in square brackets is not explicit in the original).

The point raised here is that on the face of it, the extent of the putative damage to the husband is unclear. If the perjury had not been discovered, the husband would have had to pay the stipulated amount immediately. But the full amount cannot be considered damage that the witnesses plotted to cause, since eventually the husband may have to pay it anyway—even now, after the perjury was discovered. What, then, *should* the perjurers pay? Clearly, there *is* damage, of two kinds: First, in that he would have had to pay immediately, rather than after some (indefinite) time; and second, in that perhaps in the end he might really not have to pay anything—if the wife predeceases him. Thus the damage has two components: *impatience* and *uncertainty* (or *risk*). To be sure, modern economics recognizes such questions and is able to deal with them, at least in theory. But in the 2000-

¹An ancient document that forms the basis for Jewish religious, criminal, and civil law. It consists of the *Mishna*, put into definitive form about 1,800 years ago, which sets forth the basic rules; and the *Gemara*, put into definitive form some two or three hundred years later, which discusses the Mishna and expands on it. These two parts form the nucleus of an enormous literature that has been evolving ever since, and continues to evolve to this day. In most editions of the Talmud, passages of the Mishna are intertwined with the corresponding passages of the Gemara; selected medieval (and later) commentaries appear as extensive marginal notes; and selected additional commentaries appear as extensive end notes. There are altogether sixty “tractates” or books, usually bound in twenty separate folio volumes, taking up a linear meter of bookcase space. Whole libraries are filled with thousands of additional volumes of commentaries.

²The passage sets forth what the law is if such testimony should be rendered in court.

³The marriage contract, concluded at the time of the wedding, provides a fixed sum to the wife, payable by the husband or his estate, if the marriage should be terminated during her life, either by divorce or by his death.

⁴The word “Mishna” is used both for the entire text on which the Talmud is based, and for specific passages dealing with particular issues. Similar ambiguities occur in many languages; one may say “my son studied law” as well as “yesterday Congress passed a law.”

year old Talmud, it is remarkable that the question is at all recognized—all the more so, answered. Indeed, the passage continues as follows:

“One estimates how much a person would pay for [her rights under] her marriage contract, [taking into account that she will only be paid] if she is widowed or divorced, but that if she dies [before the husband], her husband is her heir [and the perjurers pay that estimate]” (op. cit.).

This appears to solve both components of the difficulty with aplomb. An estimate is made of the value—presumably market value—of this asset, in the absence of an immediate divorce. The amount that the perjurers must pay is the damage they plotted to cause the husband: i.e., the face value of the contract (the amount to be paid in case of an immediate divorce), less the amount of this estimate (the current value of his obligation in the absence of an immediate divorce)⁵.

So far, so good. But in interpreting the Mishna a century later, the Gemara complicates matters considerably: “How does one estimate? Rabbi Khisda says, in accordance with the husband; Rabbi Nathan ben Oshaya says, in accordance with the wife” (op. cit., in the Gemara).

Most commentators on the Talmud had considerable difficulty with this passage (see below). Just what does “in accordance with the husband (or wife)” mean? But in terms of modern theories of risk-bearing, the passage becomes beautifully clear. Evidently, we are estimating not market value, but subjective value; an amount of money such that a person would as soon have the risky asset as that amount⁶. In Rabbi Nathan’s opinion, the calculation is made as above, but substituting the wife’s subjective evaluation of the contract for the market value. In Rabbi Khisda’s opinion, one uses the husband’s subjective evaluation; i.e., the maximum amount of money that the husband would pay to get rid of his obligations under the contract. Under

⁵The impatience component is dealt with also in the following passage: “We testify against John Doe that he owes his friend 1000 zuz, with payment due in 30 days;’ but he [Doe] asserts that the payment is due in 10 years. Then [if the testimony is false] one estimates how much more a person would pay for a 1000-zuz note due in 30 days than for one due in 10 years [and this is the amount to be paid by the perjurers]” (Makkoth 3a, in the Mishna near the bottom of the page). From this it is clear that the Talmud was well aware of impatience, in spite of an absolute prohibition against payment of interest for loans.

⁶Subjective value is usually less than market value. Indeed, if the price of an asset were the same as its subjective value to the buyer, the buyer would be indifferent between buying it and not buying it. But clearly, in most transactions, both seller and buyer distinctly *prefer* that the transaction be consummated.

risk aversion, the two amounts are very different, even if husband and wife have identical utility functions. The wife will be willing to accept less than the actuarial value in exchange for the contract; the husband will be willing to pay more than the actuarial value to get rid of his obligations.

In symbols⁷, denote the random variable⁸ consisting of payoffs to her by \mathbf{x} , let the face value of the contract be 200 zuz⁹, and let f be the function that associates with each risk its certainty equivalent¹⁰. It is convenient to normalize by supposing that the husband has set aside 200 zuz in a separate account, for the specific purpose of paying her the amount specified in the marriage contract, if and when it should become necessary¹¹; thus if she dies before him, he “inherits” the 200 zuz. If the witnesses had been successful in their plot, he would have been left with nothing; as it is, he is left with $200 - \mathbf{x}$. It is this amount of $200 - \mathbf{x}$ of which the witnesses wanted to deprive him. The certainty equivalent of this amount—what it is worth to *him*—is $f(200 - \mathbf{x})$, and that is what Rabbi Khisda says that the witnesses should pay.

On the other hand, the certainty equivalent of what she will receive—now that the perjury has been discovered—is $f(\mathbf{x})$. If the witnesses had been successful, she would have gotten 200; now that they are not, what she gets is worth $f(\mathbf{x})$ to her. Thus what *she* stood to gain from the perjury is $200 - f(\mathbf{x})$, and that is what Rabbi Nathan says that the witnesses should pay. Note that if the utility function u that underlies the definition of f is strictly concave—i.e., if the protagonists are risk averse—then

$$(1) f(200 - \mathbf{x}) < 200 - f(\mathbf{x});$$

that is, Rabbi Nathan requires a larger payment from the witnesses than does Rabbi Khisda.

A numerical example is perhaps in place. Suppose that both husband and wife are indifferent between a sure payoff of 80 and a lottery whose outcome

⁷Readers unfamiliar with mathematical notation and jargon may skip the next two paragraphs, continuing with “A numerical example ...”.

⁸In this paper we are interested mainly in uncertainty, and so will henceforth ignore the time factor.

⁹The zuz was a unit of currency at the time the Talmud evolved. Two hundred zuz is, in most cases, the statutory minimum for a marriage contract.

¹⁰If $u(t)$ is the von Neumann-Morgenstern utility of t zuz, and \mathbf{z} is a random variable whose values are expressed in zuz units, then $f(\mathbf{z}) = u^{-1}(Eu(\mathbf{z}))$.

¹¹Indeed, it was customary to set aside a piece of real estate—usually a field—for this purpose.

is 200 or 0 with $1/2 - 1/2$ probabilities. Thus both are risk averse, to the same degree: each is willing to pay a “premium” of 20 zuz to avoid the uncertainty of the lottery. As above, assume that the husband has set aside a piece of property worth 200 to pay the wife’s contract in case that that should become necessary. Let the probability be $1/2$ that the marriage will be terminated before the wife dies. Before the false testimony, the positions of the husband and wife are identical: both will receive 200 with probability $1/2$, and 0 with probability $1/2$. So the husband’s position is worth 80 to him, and the wife’s position is worth 80 to her. If the false testimony had been accepted, the property set aside by the husband would have been given to the wife, making his position worthless. The wife, on the other hand, would have gotten the full value of the property, namely 200. In terms of certainty equivalents, the husband would have lost 80, while the wife would have gained $200 - 80 = 120$. Rabbi Khisda says that the perjurers must pay “according to the husband,” namely 80; Rabbi Nathan, that they must pay “according to the wife,” namely 120.

More precisely, one should perhaps take cognizance of the fact that husband and wife may have different utility functions u_h and u_w , with correspondingly different certainty-equivalent functions f_h and f_w . Rabbi Khisda and Rabbi Nathan then say that the witnesses must pay $f_h(200 - \mathbf{x})$ and $200 - f_w(\mathbf{x})$ respectively; and if both u_h and u_w are strictly concave, then the latter is always larger than the former. Note, however, that the disagreement between Rabbi Khisda and Rabbi Nathan does *not* hinge on a possible difference between their utility functions; as the above example shows, the two Rabbis differ even when husband and wife have the *same* utility function.

Two difficulties, one textual and one conceptual, remain.

The textual difficulty is that the Mishna specifically refers to an estimate of how much “a person” would be willing to pay for the asset. This implies that the discussion is about a neutral third party, rather than the specific husband and wife before us. The conceptual difficulty is that while the opinion of Rabbi Khisda seems natural enough, it is difficult to understand why Rabbi Nathan relies on the subjective evaluation of the wife, who is really not a party to the litigation between the witnesses and the husband. “Ye shall do unto him as he plotted to do unto his brother.” This plot was directed against the husband; *he* would have had to pay if it had been successful, not she. Why, then, should *her* subjective evaluation be relevant?

The two difficulties can be cleared up simultaneously. Neither Rabbi Khisda nor Rabbi Nathan refer to the specific flesh-and-blood husband and

wife before us. The estimate appertains to how an outside, neutral party who has a “typical” degree of risk aversion would evaluate the risky assets $200 - \mathbf{x}$ and \mathbf{x} respectively. Thus we must indeed use a single function f as in (1), rather than separate functions f_h and f_w . The words “according to the husband” and “according to the wife” refer only to the formal places of the husband and wife in the scheme, rather than to their individual utility functions (which it would, indeed, be almost impossible to estimate). They are, in fact, merely a convenient way of referring to the two sides of inequality (1).

In these terms, the conceptual difficulty also disappears. Rabbi Khisda hews more closely to the letter and spirit of the Mosaic rule, since the husband’s loss would have been $200 - \mathbf{x}$, and it is the certainty equivalent of this that the witnesses must pay. But Rabbi Nathan hews more closely to the text of the Mishna, which states specifically that the amount to be estimated is \mathbf{x} , not $200 - \mathbf{x}$.

While we cannot be sure why the Mishna was in fact phrased in this way, one possible reason has to do with moral hazard. People might be loath to purchase the husband’s position, since doing so would decrease his disincentives to divorce her. Even the gedanken-experiment in which the court estimates how the husband himself evaluates his position is beclouded by moral hazard. Since there are conceptual difficulties in evaluating the husband’s position, Rabbi Nathan “rolls over” the estimate to the other side, a procedure not uncommon in Talmudic law.¹²

More convincing, perhaps, is a suggestion of Professor Michael Keren¹³, based on fundamental principles of criminal law. Rabbi Khisda and Rabbi Nathan agree that “the punishment should fit the crime,” but differ on whether to evaluate the crime by its effect (or putative effect) on the victim or on the perpetrator. If punishment is “retribution,” then the yardstick should be the effect on the victim; if “deterrent,” then it should be the effect on the perpetrator. The “retribution” philosophy would dictate that it is

¹²For example: No allegation can be established in court without the testimony of at least two witnesses. In criminal cases, one witness is as good as none. But in civil cases, if a single witness testifies against, say, the defendant, then the defendant must pay up, or swear a solemn oath that the witness is lying.

Suppose, now, that the defendant has a criminal record that prevents his oath from being acceptable in court. Then the oath is “rolled over” onto the plaintiff; if he swears that the witness is right, then the defendant must pay up.

¹³Privately communicated.

the putative damage to the husband that counts, in accordance with Rabbi Khisda; the “deterrent” philosophy, that it is the putative benefit to the false witnesses. And while we do not know the motives of the witnesses, it is the wife who stood to gain from their testimony; in a sense, they were acting on her behalf, even if it was without her consent. Therefore it is *her* putative gain that should determine the punishment, which is in accordance with Rabbi Nathan.

A final comment has to do with why the Gemara was not satisfied with the obvious interpretation of the Mishna, in terms of market value, described above. Again, there are two reasons, one textual, one conceptual.

Textually, the Mishna uses the word “omdin,” which generally means a rough, subjective estimate. For the more accurate objective assessments that are possible in cases of market value, the Mishna generally uses “shummin”¹⁴. Conceptually, the Mosaic injunction quoted above is best implemented by a subjective certainty equivalent; market value seems pretty irrelevant to the spirit and letter of this injunction.

In conclusion, there seems to be little doubt that fairly sophisticated concepts of the modern theory of risk bearing underlie this Talmudic passage, and were well understood by its authors.

A Glimpse at the Medieval Commentary

Most commentators, unfamiliar with modern theories of risk-bearing, had considerable difficulty with this passage. Rabbi Isaac Alfasi, living in Spain about a millenium ago, wrote, “Rabbi Khisda says, ‘in accordance with the husband:’ Is he old, and so more likely soon to die than a young man? Is he ill or well? Does he have property, which would enable him to divorce

¹⁴The difference between “omdin” (one estimates) and “shummin” (one assesses) is beautifully illustrated in the following Mishna: “He who causes bodily injury is liable on five counts: for permanent disability, pain, medical expenses, temporary disability, and humiliation. Permanent Disability: suppose he took out his [the victim’s] eye, cut off his hand, or broke his leg; one considers him [the victim] as if he were a slave to be sold in the market place, and assesses (“shummin”) how much he was worth [before the incident] and how much he is worth [now]. Pain: suppose he burned him with a spit or a nail, even on his fingernail, where no wound is caused; one estimates (“omdin”) how much a man like the victim would take in return for suffering such pain [voluntarily]. Medical expenses: . . .” (Op. Cit., Baba Kama 83b, in the Mishna).

her and give her the amount stipulated in the marriage contract¹⁵? Are they on poor terms, so that he has an incentive to divorce her? Whereas Rabbi Nathan says the opposite: Is *she* ill or well? Is *she* old or young? ..." (Alfasi Makkoth 1b).

Commenting¹⁶ on this passage from Alfasi, Nachmanides (Spain, about 800 years ago) objects that it makes no sense to look only at the husband's condition, or only at the wife's; that a reasonable estimate of the situation must take into account the conditions of both husband *and* wife. Though the remainder of his comment is not entirely transparent, it does include the assertion that "a person does not want to endanger his money by buying risks, unless he can do so for very little money"¹⁷—a succinct statement of the principle of risk aversion.

The greatest of all commentators—Rashi (Rabbi Shlomo Yitzchaki)—living in France about 900 years ago, wrote as follows:

"... It is possible to understand our Mishna in two ways. How?

Well, both the man and the woman have a doubtful privilege¹⁸ in this contract. She expects that if he dies or divorces her she will get the entire amount; and he expects that if she dies during his lifetime, he will inherit¹⁹ her rights... . So this is the meaning of it: Does one estimate *her* doubtful privilege, i.e., how much a person would pay for the benefit of her enjoyment,²⁰ this being what they [the false witnesses] will not pay, and they will pay the remainder [up to the face value of the contract]? Because the remainder is the loss they would have caused by their testimony, since even now, when their testimony has been discredited, he would gladly give her this amount for her rights.

Or, does the Mishna say that we should estimate *his* doubtful privilege, and this is the amount that the witnesses will pay? thus we would not obligate the witnesses to pay so much, i.e., the face value less the benefit of her enjoyment ..." (Makkoth 3a, in the Rashi).

¹⁵The husband's obligations under the marriage contract often constituted a significant barrier to divorce.

¹⁶In Milchamoth Hashem ("The Wars of the Lord"), Alfasi Makkoth 1a.

¹⁷"Ein adam rotzeh lessaken bemamono likach sfekot ela bedamim kalim" (op. cit.).

¹⁸"Z'chut sahek;" literally, the privilege of the risk.

¹⁹By having the husband write off the face value of the contract immediately (at the time of the marriage), Rashi has normalized so that all payoffs to all protagonists are non-negative. If the wife dies before the husband, the amount of the contract appears as a credit to him; he "inherits" it. This normalization is like in the body of the paper.

²⁰"Tovat Hana'a."

Rashi is rarely so expansive; usually his writing is terse and to the point. His expansiveness in this instance may indicate that he is feeling his way on unfamiliar ground, or at the least, expounding an unfamiliar viewpoint. Be that as it may, it appears that Rashi may have come fairly close to the interpretation in the body of this paper.