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**MANIPULATING ALLOCATION JUSTICE:
HOW FRAMING EFFECTS CAN INCREASE THE
PREVALENCE OF THE TALMUDIC DIVISION
PRINCIPLE "SHNAIM OHAZIN"**

by

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Manipulating Allocation Justice: How Framing Effects can Increase the Prevalence of the Talmudic Division Principle "Shnaim Ohazin"

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Abstract

In the role of judges in bankruptcy problems, people may prescribe various just divisions of the available goods to claimants who have rights for them. Two widespread division rules are equality and proportionality. A less known rule is the Talmudic "Shnaim Ohazin" principle, whose basic rationale is applying an equal division only to that part of the goods which is genuinely under dispute. This paper demonstrates that the ratio of subjects that prefer the "Shnaim Ohazin" principle over equality and proportionality can be increased by a simple framing manipulation. These results suggest that framing effects might be a prevalent factor in the realm of distributive justice.

Introduction

This paper focuses on choices of division rules for the purpose of goods allocation in the case of two claimants and a sum of available goods which is smaller than the sum of the justified demands. Such cases belong to the sphere of distributive justice and are often referred to as bankruptcy problems. In general, resolution of a bankruptcy problem can be described as a transformation of inputs, such as demands, rights or needs of the recipients, to outputs, which are the final allocated shares. This transformation usually requires a division rule, according to which the goods are allocated to the parties involved.

Here, we consider three division rules: equality, proportionality and a less known Talmudic principle named "Shnaim Ohazin" (to be referred to as SO). The equality rule guarantees an equal share of the goods to each recipient, regardless of differences between inputs, in case such exist. The widespread proportionality rule is often used when such differences are to be taken into account. According to this rule, the same proportion has to be maintained between the inputs and the outputs. An alternative to the equal and proportional divisions is the SO principle, which appears in the Babylonian Talmud¹. The name of this rule, "Two hold (a garment)", refers to a Mishna² which describes the following division problem and its solution:

"Two hold a garment... if one of them says, "It is all mine" and the other one says, "Half of it is mine",... the former then receives three quarters and the latter receives one quarter" (Baba Mezi'a, Fol. 1, Epstein (ed.), 1935, quoted in O'Neal, 1982).

¹ A collection of writings constituting Jewish civil and religious law written down around the 2nd-5th centuries A.D.. It consists of Mishna (the basic text) and Gemara (interpretation and discussion of the Mishna).

² The word "Mishna" can also refer to single teaching on a specific issue in the Mishna text.

The rationale of this allocation is an equal division of the disputed part of the goods. Since there is no dispute over the half of the garment which is claimed only by the first person, that person alone gets it, and the remaining half claimed by both is divided equally between them, resulting in an overall $\frac{3}{4}:\frac{1}{4}$ allocation. Aumann and Maschler (1985) described this principle as the rationale behind several Talmudic divisions and showed that those divisions coincide with the nucleoli of the corresponding coalitional games. Previously, the problem of the garment and the solution of Rabbi Abraham Ibn Ezra for a similar problem were discussed in O'Neal (1982).

As opposed to the equal and proportional rules, it seems that the SO principle has received little to no attention in the psychological literature. In this paper we will try to propose preliminary research about the psychological reality of SO. The main theoretical framework of the paper is the study of heuristics and the influence of framing on problem solving and decision making. Tversky and Kahneman (1981) demonstrated that formulation of decision problems influence their subjective perception and the decision maker's preferences. In regard to those findings, the central concern of the current paper will be to examine whether a framing manipulation can cause subjects to shift from the "standard" proportional and equal divisions to the SO division. Such concern might have consequences for a more general issue, namely the relevance of the framing paradigm to the sphere of distributional justice, adding to previously factors such as convention and context.

Problem Definition

A problem of bankruptcy with two recipients and a sum of legitimate claims which exceeds the available sum of goods can be formalized as following: E is the sum total of the goods, while d_1 and d_2 are the inputs (in this case demands). Accordingly, $d_1 \leq d_2$ and $E < d_1 + d_2$. E is then to be divided into two outputs (shares) a_1 , a_2 using some division rule, so that $E = a_1 + a_2$. As mentioned earlier, unequal rights or legitimate claims are only one of several possible differences between the recipients. They belong to the first of six such categories proposed by Bar-Hillel and Yaari (1984). The working assumption here is that the recipients are equal in the benefit they receive from the goods. In other words, any part of E has the same value for both recipients. This assumption does not apply for categories 4-6 of Bar-Hillel and Yaari: differences in needs, tastes and beliefs (Bar-Hillel & Yaari, 1993).

Three Division Rules

The first division rule we consider is equality. According to this rule, the goods are divided equally between the recipients. The equality rule reflects what Eckhoff (1974) calls "objective equality", which does not take into account different demands or any other differences between the recipients. This rule is also a specific case of the proportional rule, when the relation between the demands is 1 (equal demands). The

allocation using this rule is: $a_1 = \frac{E}{2}$, $a_2 = \frac{E}{2}$.

The second rule is proportionality³. The proportional rule is central to Adam's equity theory (1965) and was extensively discussed by Homans (1961) and Walster, Walster and Berscheid (1978). The basic rationale of this rule is guaranteeing an equal share in relation to each demand, so that $\frac{d1}{d2} = \frac{a1}{a2}$. $a1$ and $a2$ can be derived from the

$$\text{formula: } a1 = \frac{d1}{d1 + d2} \times E, \quad a2 = \frac{d2}{d1 + d2} \times E.$$

The third division principle is SO. According to this principle, each of the recipients explicitly withdraws any claims on the part of the goods which he does not demand, while the disputed part is divided equally between the two. For each sum of goods and pair of demands there is a single division which is compatible with the SO principle. SO might also be regarded as the sum of two proportional subdivisions: first, of the non-disputed part claimed only by one of the recipients, and second, of the remaining disputed part claimed by both recipients. In this sense, suggesting a solution which corresponds with SO does not necessarily mean applying a distinct division principle, but rather interpreting the allocation problem in a different way. This duality of SO may have implications on its psychological relevance, which will be discussed further on. The formula of SO for two recipients is:

$$a1 = \frac{E - \max\{(E - d1), 0\} + \max\{(E - d2), 0\}}{2}, \quad a2 = \frac{E + \max\{(E - d1), 0\} - \max\{(E - d2), 0\}}{2}.$$

Note that for cases of two recipients, SO division matches the Shapley value (Shapley, 1953) of a coalitional game⁴ with the following coalitional function: $v(S) = \max\{(E -$

³ Also known as Adam's law, equity rule, relative equality principle and others.

⁴ A coalitional game is formally defined by the pair $(N; v)$, where $N = \{1, \dots, n\}$ is the set of all players and the function $v(S)$, which associates a value with the a coalition S for every $S \subseteq N$.

$\Sigma d_i, 0\}$ (Σd_i relates to the recipients which are not in group S). It also corresponds to O'Neal's rule and other divisions discussed in O'Neal (1982).

Experiment

In order to examine the psychological relevance of the SO principle, an experiment was designed, in which judgments in relation to a problem of distributive justice were elicited. Respondents were introduced to a hypothetical bankruptcy problem and asked to propose a fair allocation of the disputed goods. The posed question was open-ended and respondents were asked to explain their solution.

In the problem, real estate property was to be divided in a just manner between two claimants. Similarly to the Talmudic garment problem, where one wants it all and the other wants a half. The numerical values were set to 12 units for the value of the goods, while one recipient claims 6 units, and the second recipient claims the entire 12 units. The chosen values lead to distinct solutions for the equal, proportional and SO divisions, and all three solutions are in integers and relatively easy to calculate. The experiment included two formulations of the problem. In the first formulation the property consists of 12 apartments. The first claim is 6 apartments and second claim is 12 apartments. In the second formulation the property consists of 6 apartments and a villa with same value as the 6 apartments, while the first person claims the 6 apartments and second person claims the 6 apartments and the villa. Thus, the difference between the two versions was replacement of six units (six apartments) in the first formulation with one unit of a different kind but same worth (a villa) in the second formulation.

At the heart of the experiment was an attempt to cause a perceptual separation between the disputed and undisputed parts of the goods. It was then examined whether such a manipulation would lead respondents to propose the SO solution more frequently.

Method

Participants and Procedure: 80 respondents aged between 20 and 60 participated in the experiment. About a third of the participants were acquaintances who filled the questionnaire via e-mail. Another third were undergraduate students of the Cognitive Science department at the Hebrew University of Jerusalem who received the forms at the break in a lecture. The remaining third were students of the Hebrew University recruited in various locations at the university campus as well as colleagues of relatives not affiliated with the university. The respondents were randomly divided in two groups. Each group responded to one of the experiment's formulations. Filling of the questionnaire was not limited in time. Usually, an answer took about 2-5 minutes. No monetary or other rewards were given to the participants.

First Formulation

A real estate firm went bankrupt due to irregularities in its conduct. Following the bankruptcy, 12 flats with equal monetary value were left. One example of the misconduct of the firm is that they sold 6 out of the 12 flats to person A and all the 12 flats to person B. When both persons arrive to receive their property, person A demands only 6 apartments, according to his contract with the firm, while person B demands all 12 apartments, according to his contract with the firm.

What would be, in your opinion, a fair allocation of the property between the claimants?

Person A should receive _____

Person B should receive _____

Explain your answer (not mandatory):

Second Formulation

A real estate firm went bankrupt due to irregularities in its conduct. Following the bankruptcy, 6 flats with equal monetary value and one villa were left. The monetary value of the villa equals the value of the 6 flats taken together (6 times more than a single flat). One example of the misconduct of the firm is that they sold 6 flats to person A and the same 6 flats to person B, who also purchased the villa. When both persons arrive to receive their property, person A demands only the 6 apartments, according to his contract with the firm, while person B demands the 6 apartments and the villa, according to his contract with the firm.

What would be, in your opinion, a fair allocation of the property between the claimants?

Person A should receive _____

Person B should receive _____

Explain your answer (not mandatory):

Results

	First Formulation 6;12			Second Formulation 6; 6 & villa		
	N=40			N=40		
Allocation	Equal (6;6)	Proportional (4;8)	SO (3;9)	Equal (6; villa)	Proportional (4;2+villa)	SO (3;3+villa)
Number of Participants	7	27	6	6	19	15
% of all the participants	17.5	67.5	15	15	47.5	37.5
% of participants who considered differences between demands		82	18		56	44

Additionally to the data presented in the table, in the first formulation, three participants did not give a numerical answer and suggested allocating the property according to the "first come, first served" principle. In this case, whoever signed the contract first, ought to receive his demand fully. In the second formulation, two participants suggested an allocation of 5 flats for the first recipient and the villa and one flat for the second. Both did not supply grounds for their proposal. Another three people claimed this issue can only be solved in court, and two others allocated more property than was available.

Z proportion test $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$ was used to determine whether SO rates

differed significantly between the formulations. Though remaining a minority, the percentage of the participants who suggested SO allocation in the second group was significantly higher than the percentage of participants who suggested the same allocation in the first group ($Z=2.287$, $p<0.05$). A similar calculation which does not include participants who chose equal division shows a significant difference between the groups as well ($Z=2.294$, $p<0.05$).

Interpretation

The difference between the two groups can be interpreted in (at least) two ways. The first interpretation focuses on a possible gap in the computational difficulty between the two formulations. In the second formulation, an additional calculation of dismantling the villa into 6 units, which were then to be summed up with the other 6 units, was needed in order to allocate the goods according to the proportional rule. Putting the villa aside, and thus avoiding the harder calculation, might have led some participants to suggest an allocation which matched the SO principle.

The second interpretation has to do with a psychological difference in the perception of the problem. It is possible that the replacement of 6 apartments with a villa caused some participants of the second group to regard the second formulation as a problem in which an allocation which matches the SO division is fairer than other possibilities and especially than proportionality.

This does not necessarily mean that people used different allocation rules for the same problem, but rather that people used the same rule differently because they conceived the different formulations as different problems. More specifically, participants might have regarded the first formulation as a single allocation problem, whereas the second formulation might have been perceived as a problem which can be divided into two smaller problems. Under this perspective, in both formulations the proportional rule was used. In the first formulation it was used once, for the demands 6 and 12, whereas in the second it was used twice: first time for the demands 6 (the value of the villa) and 0, and second time for the demands 6 and 6. This leads us back to the two ways in which the SO principle can be viewed: as a division rule and as a division strategy of partitioning the problem and applying for each part the proportional rule. These two views are not necessarily different, since as mentioned, equal division of the disputed part of the goods can be a specific case of proportional division in which the demands are equal. Either way, the relative success of the perceptual manipulation demonstrates the relevance of the Tversky and Kahneman framing paradigm to the sphere of distributive justice.

Future Research

The existence of two interpretations for the above presented results may raise a problem in the experiment's design. It is possible to identify in the independent variable two variables: the category of the units (which changed from flat to villa) and the conversion of six objects to one object with an equal value.

One solution for this problem can be allowing explicit identifiably, using numeration of the objects belonging to a property which consists of the same objects in both scenarios. The independent variable in this case will be the existence of specific numbers associated with the claimed objects of the property. Accordingly, in the first formulation recipients will present general claims regarding the property (e.g. 6 apartments) while in the second formulation they will be claiming for specific objects of the property (e.g. apartments No. 2 to No. 7).

A second solution would be a stronger manipulation, so that in both scenarios half of the objects will belong to one category, whereas the second half will belong to another category (e.g. 6 apartments and 6 duplexes, all with same monetary value). In the first formulation, the demands will be general, concerning a certain amount of the property (e.g. 12 pieces of real estate), while in the second formulation demands will be specific (e.g. 6 duplexes and 6 apartments).

In case one of the manipulations would reveal a significant difference between the two formulations, the possibility that the difficulty of computation played a role in the presented experiment could be excluded.

In the two aforementioned designs the main manipulation focused on the frame of the problem. A different strategy for a follow-up experiment would be an explicit survey of the preference rates of the SO division for a similar scenario, but without a framing manipulation. In such a survey, the first scenario of the experiment will be presented to the subjects, who will then be asked to choose between several divisions. Each of the proposed divisions will be followed by a brief description of their rationale. This survey might provide an additional index for the psychological relevance of SO as a principle

which maintains distributional justice. The explicit multiple choice strategy can also be combined with one of the suggested framing manipulations, which enhance the separation between the disputed and undisputed parts of the property. One could expect that such combinations might lead to higher rates of SO preference than in the presented experiment, since part of the participants who would otherwise propose a proportional division, might now be convinced by the SO rational.

Conclusion

There are many division principles, three of which were discussed in the current paper. The justification for using a certain principle might involve various factors, such as social, cultural and ideological conventions regarding the concept of justice as well as the specification of the problem. This paper suggests the framing of the problem as an additional parameter which might play a role in the choice of a just division rule and subsequent allocation. Additionally, it has been demonstrated that in some cases, the SO principle might be relevant for the human justice apparatus.

The presented initial results leave many open questions, some of them regarding the nature of the psychological relevance of SO and others its extent. Nevertheless, they might provide a fertile ground for future psychological research on distributive justice in general and SO in particular.

References

- Adams, J. S. (1965). Inequity in social exchange. In Berkowitz, L. (ed.). *Advances in Experimental Social Psychology*, 2, 267-299.
- Aumann, R. J. & Maschler, M. (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory*, 36(2), 195-213.
- Bar-Hillel, M., & Yaari, M. (1993). Judgments of distributive justice. In Mellers, B. A. & Baron, J. (eds.), *Psychological Perspectives on Justice*, 56–84. New York: Cambridge University Press.
- Eckhoff, T. (1974). *Justice: Its Determinants in Social Interaction*. Rotterdam: Rotterdam Press.
- Epstein, I. (ed.). (1935-48). *The Babylonian Talmud*. London : Soncino Press.
- Harris, R.J. (1976). Handling negative inputs: On the plausible equity formulae. *Journal of Experimental Social Psychology*, 12, 194-209.
- Homans, G. C. (1961). *Social Behavior: Its Elementary Forms*. New York: Harcourt, Brace, Jovanovich.
- O’Neal, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences*, 2, 345-371.
- Shapley, L. S. (1953). A value for n -person games. *Annals of Mathematics Studies*, 28, 304-317.
- Tversky, A. & Kahneman D. (1981). The framing of decisions and the psychology of choice. *Nature*, 211(4481), 453-458.
- Walster, E., Walster, G. W. & Berscheid, E. (1978). *Equity Theory and Research*. Boston: Allyn and Bacon.

Yaari, M. & Bar-Hillel, M. (1984). On dividing justly. *Social Choice and Welfare*. 1,

1-24.